

# Reliability evaluation using variation mode and effect analysis: Application to CorPower's mooring pre-tension cylinder

Pär Johannesson, Thomas Svensson, and Hervé Gaviglio

**Abstract**— This paper presents a case study of a fatigue design assessment of a critical component in the CorPower C3 half-scale prototype wave energy converter. The aim is to evaluate a design case for the structural reliability of the pre-tension cylinder subjected to internal pressure, which through the piston rod connects to the mooring. The fatigue design calculations are based on the pressure vessel standards that has predetermined safety factors. The reliability assessment is re-evaluated by means of the probabilistic Variation Mode and Effects Analysis (VMEA) tool where relevant uncertainty sources are identified. Each source is characterized by its size and sensitivity to the fatigue load-strength measure. All uncertainties are then combined into a reliability measure in terms of a total uncertainty that can be converted into a safety factor for design. In this case, the uncertainty assessment shows that three uncertainty factors connected to the fatigue strength (scatter, notch factor and fatigue model) dominates the total uncertainty. The safety requirements based on the standard and on the VMEA are evaluated, compared and discussed. Furthermore, the VMEA result will feed into the condition monitoring and maintenance strategies. The bottom line is that the detailed probabilistic evaluation using VMEA gives better ground for improvement work, design updates and maintenance planning, compared to the standard.

**Keywords**—marine energy, wave energy converter, pressure vessel standard, fatigue, variation mode and effect analysis, load analysis, condition monitoring

## I. INTRODUCTION

MARINE energy devices operate in harsh environments but still need to perform reliably and produce the expected amount of energy, which give rise to huge engineering challenges. Reliability has been identified as a key issue for the successful

development of the marine energy sector, [1]-[2]. The RiaSoR (Reliability in a Sea of Risk) project addresses this strategic industrial need for guidance in reliability design. The RiaSoR 1 project developed a reliability framework, [3], building on established practices from the automotive industry, by implementing and adapting the Variation Mode and Effects Analysis (VMEA) methodology to ocean energy applications. The RiaSoR 2 project builds on the reliability framework developed in RiaSoR 1, aiming at further developing the methodologies, especially considering fatigue assessment, load uncertainty assessment and condition monitoring, enabling more refined designs and efficient maintenance of wave energy converters (WECs).

The goal of this paper is to demonstrate methodologies for reliability evaluation and data processing for the monitoring framework. The methodologies are presented in the form of a case study that evaluates a design criterion for the fatigue reliability of the pre-tension cylinder in CorPower Ocean's C3 half-scale prototype WEC. The pre-tension cylinder is subjected to internal pressure and the required wall thickness is assessed for fatigue strength design case. The design calculations are based on the pressure vessel standards, [4], which provides recommended safety factors for design. The case study re-evaluates the reliability assessment using Variation Mode and Effect Analysis (VMEA) that is presented in detail below. The purpose of the re-evaluation is firstly to compare the VMEA methodology to the reliability design according to the pressure vessel standard regarding the safety judgements, and secondly to investigate if the VMEA evaluation gives a better ground for improvements and updating, compared to the standard. Additionally, by monitoring the loads in service, the residual life can be estimated, which connects to the condition monitoring framework and maintenance planning.

## II. THE CORPOWER SYSTEM

The CorPower WEC addresses the key challenges of efficient wave energy harvesting in a unique way. The WEC is of point absorber type, with a heaving buoy on the surface absorbing energy from ocean waves and which is connected to the seabed through a mooring system. CorPower's lightweight design and inherent

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P. Johannesson and T. Svensson is with RISE Research Institutes of Sweden, Gibraltargatan 35, SE-412 79 Göteborg, Sweden (e-mail: thomas.svensson@sinnesro.org).

H. Gaviglio is with CorPower Ocean, Brinellvägen 23, SE-114 28 Stockholm, Sweden (e-mail: herve.gaviglio@corpowerocean.com).

ability to survive storms are enabled by two major sub-systems:

- The WaveSpring phase control technology. Phase control allows the buoy to move in resonance with incoming waves over a broad range of periods. This sub-system can be detuned to make the device more transparent to incoming waves during storms.
- The pre-tension system enables energy capture in both directions, while balancing the system at equilibrium.

CorPower's half-scale prototype, which was deployed in 2018 in the Orkney islands, is illustrated in Fig.1.

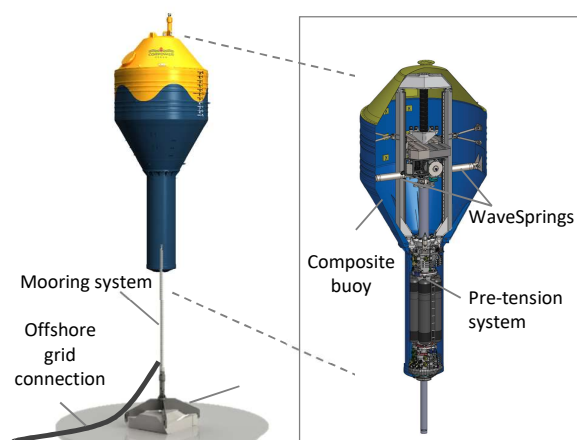


Fig. 1. CorPower's WEC.

### III. VMEA METHODOLOGY

An important goal of engineering design, and marine energy in particular, is to get a reliable device. In industry, the method of Failure Mode and Effect Analysis (FMEA) is often used for reliability assessments, where the aim is to identify possible failure modes and evaluate their effects. This is primarily a qualitative method which points out weaknesses in design, but without giving measures of the resulting reliability. Studies of FMEA have indicated that the failure modes are in most cases triggered by unwanted variation that may be quantified [5]. Furthermore, a general design philosophy within reliability and robust design methodology is to make designs that avoid failure modes as much as possible [6]-[8]. Thus, it is important that the design is robust against different sources of unavoidable variation.

Therefore, a tool for addressing robustness against variation was proposed, Variation Mode and Effect Analysis (VMEA), which takes the quantitative measures of failure causes into account based on ideas from statistics, reliability and robust design.

The VMEA method was first presented in [9]-[10] and further developed in [11]-[13]. An adaptation to marine energy applications is found in [3], while general presentations of the methodology are given in [6], [14]-[16].

The VMEA method aims to take an engineering standpoint and solving practical reliability problems using a probabilistic framework. The goal is to capture all relevant uncertainty sources, also including model uncertainties that are treated in a similar way as systematic errors in measurement uncertainty, [17]. The VMEA method represents a first-order, second-moment reliability method, see e.g. [18]-[20] for general references, meaning that the target function in question is linearized around the target value and that the uncertainties are approximated with their statistical second-moment properties, variances and covariances. In this respect it follows the ideas in [21]-[22], for instance, the JCSS reliability recommendations. Specific recommendations and applications to wave energy are found in [1]-[3], [23]-[28].

In one important respect the VMEA methodology differ from the established first order, second moment methodology. Namely, the VMEA resulting safety margin is established only for 95% reliability. This is because the input information usually is limited to justify statistical judgements in the "tail of the distribution", the estimated overall standard deviation cannot be trusted as representative for rare events. To solve this, the VMEA methodology includes a complementary additional margin, an extra safety factor based on risk judgements.

#### A. Work Process – 7 Steps

The general procedure for making a VMEA is common for all development phases, however the different steps will be adapted to the amount of information available, as will be explained later. The work process can be split into four activities "Define-Analyse-Evaluate-Improve", as illustrated in Fig. 2, and can be described in the following seven steps:

*Target Variable Definition:* The first step is to define the target variable, i.e. the property to be studied, which can be the life of a component, the maximum stress or the largest defect.

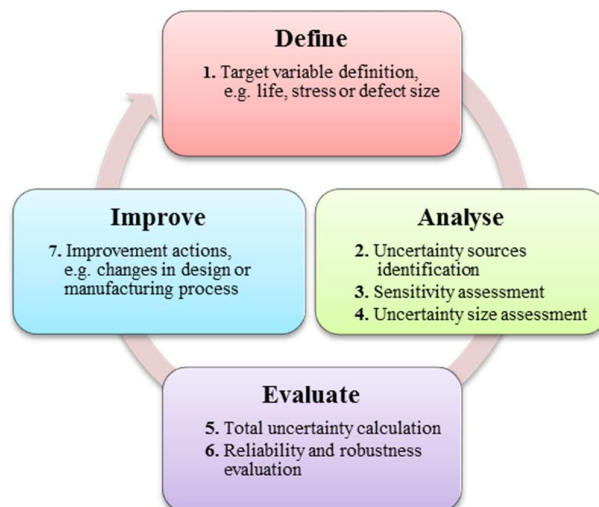


Fig. 2. VMEA in the design and improvement cycle.

**Uncertainty Sources Identification:** In this step all sources of uncertainty that can have an impact on the target variable are identified and categorised as either load or strength. The uncertainty sources may be classified as scatter, statistical and model uncertainties.

**Sensitivity Assessment:** Here the task is to evaluate the sensitivity coefficients of the sources of uncertainty with respect to the target variable by numerical calculations, experiments, previous experience etc.

**Uncertainty Size Assessment:** Here the task is to quantify the size of the different sources of uncertainty by experiments, previous experience, engineering judgement etc.

**Total Uncertainty Calculation:** The final step of the core VMEA activity is to calculate the total resulting uncertainty in the output of the target function by combining the contributions from all uncertainty sources according to their sensitivities and sizes via the aforementioned equation.

**Reliability and Robustness Evaluation:** The result of the VMEA can be used to evaluate the reliability and robustness in order to compare design concepts, find dominating uncertainties, derive safety factors etc.

**Improvement Actions:** The last important step is to feedback results to the improvement process by identifying uncertainty sources that are candidates for improvement actions and evaluate their potential for reliability improvements.

Although the core VMEA methodology is from step 2 to step 5, problem definition (step 1), reliability evaluation (step 6) and improvement work (step 7) are equally essential in the design process. Therefore, all seven steps are included in the overall VMEA methodology to cover the design and improvement cycle illustrated in Fig. 2.

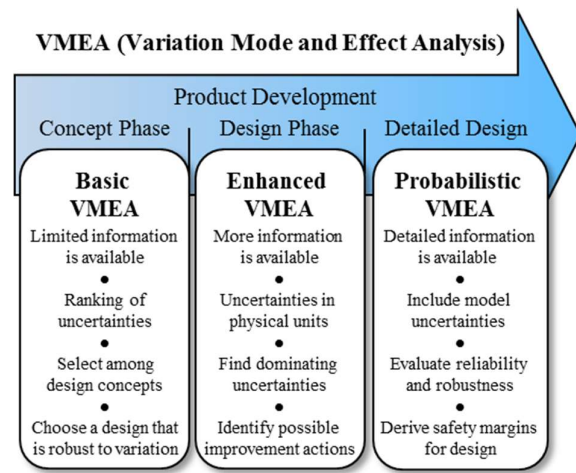


Fig. 3. VMEA in different design stages.

## B. Development Phases

The VMEA is split into three different phases as shown in Fig. 3, namely 1) basic VMEA, in the early design stage when little is known about variations, 2) enhanced

VMEA, further in the design process when the sources of variation can be better judged, and 3) probabilistic VMEA, in the later design stage when detailed information is available for variations.

### 1) Basic VMEA

In the Basic VMEA the goal is to identify the most important sources of variation and evaluate their effect. The sizes of variations as well as their sensitivities to the studied product property are evaluated on a scale from 1 to 10. The Basic VMEA can be built up from a cooperative brain storm session. It gives a qualitative picture of the uncertainties and can be used for comparison of different design concepts and prioritisation for further studies.

### 2) Enhanced VMEA

A refinement of the Basic VMEA may be done by quantifying uncertainties by judging their standard deviations by means of standard rules and judge sensitivities by fundamental physical knowledge. This analysis is called an Enhanced VMEA and can be used for a preliminary assessment of a safety factor needed for taking the studied uncertainties into account.

### 3) Probabilistic VMEA

A further refinement, called the Probabilistic VMEA, is developed by getting more information about the most critical uncertainty sources. Standard deviations are obtained by more detailed studies of empirical results. Sensitivity coefficients are found from numerical sensitivity studies or differentiation of physical/mathematical models. The result of such an analysis gives an estimate of the resulting total uncertainty and a corresponding statistical safety factor.

## C. Mathematical Principles of VMEA

The method is based on characterising each uncertainty source by a statistical standard deviation and calculating its sensitivity with respect to the target variable, such as fatigue life, maximum stress etc. The VMEA method combines these into the total prediction uncertainty, denoted  $\tau$ , which is obtained by the root sum of squares (RSS) of the uncertainties in Eq. (1).

$$\tau = \sqrt{\sum_{i=1}^n \tau_i^2} = \sqrt{\sum_{i=1}^n c_i^2 \sigma_i^2} \quad (1)$$

where  $\tau_i$  is the resulting uncertainty from source  $i$  and is calculated as the product of the sensitivity coefficient  $c_i$  and the uncertainty  $\sigma_i$  of source  $i$ . The total number of uncertainty sources is  $n$ . Note that VMEA is a so-called second-moment method since it uses only the standard deviation to characterise the distribution of the uncertainty sources.

## D. Evaluation of Reliability and Uncertainties

This probabilistic VMEA approach represents a first order, second moment reliability method. "First order" is

due to the fact that the influence of each term is approximated by one single linear term, and “second moment” is that the probabilistic influence is approximated by second moment statistics, variances and covariances.

VMEA is a probabilistic method that studies the variation and uncertainty around a nominal design. Based on all variation and uncertainty sources, the methodology determines a statistical safety distance that, together with additional engineering risk judgements, gives a proper safety factor against eventual failure. The statistical safety distance is constructed by means of a confidence interval, in turn determined from an overall standard deviation of the target function, often formulated as load/strength-function. The standard deviation, being the square root of the variance, is found using the *Gauss' approximation formula*

$$\text{Var}[f(x_1, x_2, \dots, x_n)] \approx \sum_{i=1}^n c_i^2 \text{Var}[x_i] + \text{Covariances}. \quad (2)$$

This formula gives the variance of the target function  $f$  as the sum of variance contributions from different influencing variables  $x_i$ , each described by its own variance together with its influence by means of its sensitivity coefficient  $c_i$ . Formally, the sensitivity coefficient  $c_i$  is the partial derivative of the target function  $f$  with respect to  $x_i$ , but it is often best approximated by a difference quotient. Covariances between the influencing variables also contribute, which however can usually be neglected or avoided by re-formulation of the model.

The load/strength function used as the reliability target is often defined in logarithmic scale. This usually reduces non-linearities and makes the variance representation of uncertainty more scale independent. Also, if the natural logarithm is used, the uncertainty of the logarithmic property is approximately equal to the relative uncertainty of the property itself, namely

$$\text{std}(\ln x) \approx \frac{\text{std}(x)}{\text{mean}(x)}. \quad (3)$$

Many uncertainties must be assessed by engineering judgements. Engineers are often not too familiar with statistical properties like standard deviations or variances and it is often easier to find judgements about a possible uncertainty interval; possible minimum and maximum of a certain property. From such judgements it is convenient to use the standard deviation,  $s$ , of a uniform distribution, namely

$$s = \frac{\max - \min}{\sqrt{12}}. \quad (4)$$

Now assume that the target function is formulated in logarithmic form, e.g. logarithmic load/strength difference  $\ln S - \ln L$  as will be used in the case study below. The final result by means of a standard deviation is calculated by Eq. (2), and the target function can be

approximated by a Gaussian distribution, by means of the statistical central limit theorem. Thus, multiplying the standard deviation by 1.64 gives a 90% confidence interval for the logarithmic target and its lower value represents the point of 95% reliability. By taking the exponential of the confidence interval we obtain a corresponding safety factor. The chosen limit for the statistical safety factor, representing 95% reliability, is usually not safe enough for service. The reason for this choice is that the approximations involved in the analyses makes judgements of higher percentages doubtful; the statistical theory is not trustworthy further out in the tails. Therefore, the statistical safety factor must be completed with an extra safety factor, based on non-statistical considerations, such as the amount of severity of failure, the risk of human mistakes or extreme events.

#### IV. FATIGUE STRENGTH CASE

Several presumptive failure causes of the pre-tension cylinder have been identified and investigated by CorPower Ocean. The investigations were made with respect to the demand on the half-scale prototype of survival for one year in service for the testing programme. The evaluations were based on criteria from pressure vessel standards, [4], including predetermined safety factors.

The paper presents a reliability evaluation of the case “Endurance limit calculation acc. to EN 13445-3:2014, Sect. 17. Calculation of the wall thickness of the pre-tension cylinder subjected to internal pressure.” by means of the probabilistic VMEA tool. The purpose of this re-evaluation is firstly to compare the VMEA methodology to the established reliability design tools regarding the safety judgements, and secondly to investigate if the VMEA evaluation gives a better ground for improvements and updating, compared to the standard. Additionally, the VMEA will feed into the condition monitoring and maintenance work.

#### V. PRESSURE VESSEL STANDARD

The failure mechanism to be considered is fatigue and the pressure vessel standard, [4], uses the Palmgren-Miner property “damage” as target. For the calculations the standard uses a fatigue model including an asymptote at the “endurance” limit (ANL model). The fatigue life model for unwelded areas are used, which according to the standard is as follows:

$$N = \begin{cases} \left( \frac{46000}{\Delta\sigma_{fict} - 140} \right)^2, & \Delta\sigma_{fict} > \Delta\sigma_D \\ 2 \cdot 10^6 \cdot \left( \frac{\Delta\sigma_D}{\Delta\sigma_{fict}} \right)^{10}, & \Delta\sigma_D \leq \Delta\sigma_{fict} \leq \Delta\sigma_{cut} \\ \infty, & \Delta\sigma_{fict} < \Delta\sigma_{cut} \end{cases} \quad (5)$$

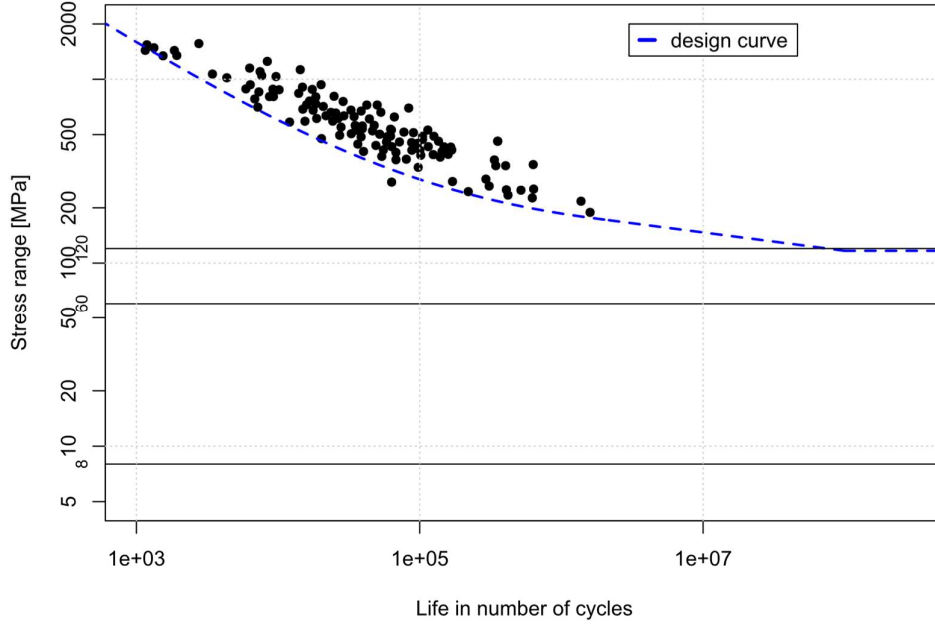


Fig. 4. Design curve (dashed blue) and validation data (black dots).

where  $\Delta\sigma_{fict}$  is the so-called *fictive stress range* and the break point values are  $\Delta\sigma_D = 172.5$  MPa and  $\Delta\sigma_{Cut} = 116.7$  MPa. The three conditions above defines the *design curve*, the dashed blue line in Fig. 4. The figure also illustrates some *validation data*, black dots, found in the comments to the standard.

#### E. Design loads

The *design loads* are derived by CorPower from three different operation cases of the equipment, and three load levels by means of the property *fictive stress range* are calculated. In this case study the loads during the design life of one year is presented in Table I, where the highest level occurs two hundred thousand times, the middle level five hundred thousand times, while the lowest level occurs two million times. These three cases have been chosen as a simplified representative loading of the structure. While they do not directly correspond to different sea states or weather conditions, they were chosen to span a large range of stresses and reflect real-life loading. In Fig. 4, the three nominal load cycle ranges representing the design load cases, and only the largest one, 120 MPa, exceeds the defined fatigue limit. Note that the assumed load spectrum presented in Table I and Fig. 4 differs from actual values so as to avoid compromising CorPower's design values.

TABLE I  
DESIGN LOADS FOR ONE YEAR

Case, $i$	Fictive stress range, $\Delta\sigma_i$ [MPa]	Number of cycles, $n_i$
1	120	$2 \cdot 10^5$
2	60	$5 \cdot 10^5$
3	8	$2 \cdot 10^6$

#### F. Damage prediction

To assess the reliability, the damage according to the Palmgren-Miner rule is calculated. Only the largest load level contributes to the damage, which becomes  $D = 0.00265$ , corresponding to a predicted life of 377 years. It can be observed that the damage is far from unity, and thus the conclusion is that the design is well within the safe region.

However, by referring to fatigue “damage” the safety margin is related to life, which is very uncertain in the actual region, both because of lack of validation data and because of the vicinity to the highly uncertain “fatigue limit”. Since possible model errors are not taken into account in the methodology of the standard, the seemingly large safety margin may be misleading.

### VI. VMEA RELIABILITY EVALUATION

#### G. Target function definition and nominal values

For the VMEA reliability analysis we choose to consider the *equivalent strength and load* instead of the calculated damage and define the target variable as the difference (in log-scale) between strength and load

$$Y = \ln S - \ln L \quad (6)$$

where  $S$  is the equivalent fatigue strength at two million cycles and  $L$  is the equivalent fatigue load range representing the target life. More precisely, the equivalent fatigue strength is the stress range corresponding to a life of two million cycles, and the equivalent fatigue load is the stress range (repeated two million times) that is damage equivalent to design loads corresponding to the target life. Note that the equivalent load and strength



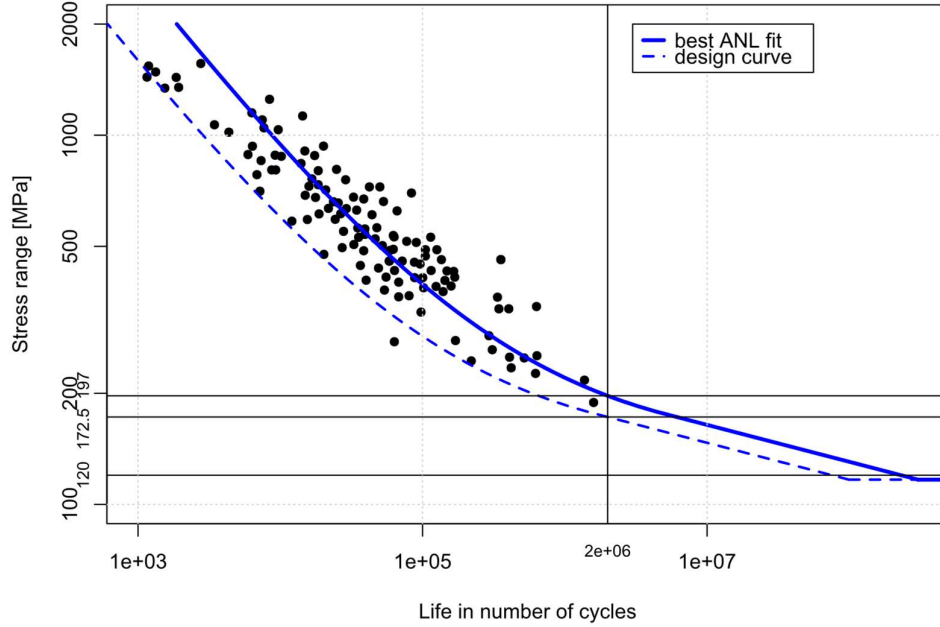


Fig. 5. Median fatigue curves (solid), design curve (dashed) and validation data (dots).

values are connected through the same number of cycles,  $n_0 = 2 \cdot 10^6$ . Failure occurs when the load becomes larger than the strength, i.e. the failure criterion is  $Y < 0$ .

#### H. Nominal strength

It is apparent that the *design curve* in Fig. 5 represents a lower quantile of the fatigue strength distribution. For the VMEA analysis we need the median curve but the standard gives no information about which quantile that is used for the design curve. Thus, we need to find the median curve, which is done by translating the design curve in order to fit to the validation data in the pressure vessel standard. A numerical search gives the best fit: multiplying the design life curve by a factor  $\gamma = 3.057$ , see the solid blue line in Fig. 5. The standard deviation of the residuals in the curve fit is  $s = 0.59$ , and the design curve is then translated  $\ln(\gamma)/s = 1.89$  standard deviations. Using a normal distribution assumption, it means that the design curve represents the 3% quantile in the distribution, i.e. 97% survival probability.

For the VMEA evaluations of reliability, the fatigue limit will be set to  $\Delta\sigma_{cut} = 0$  MPa, and thus the life model (best fit ANL model without fatigue limit) can be formulated as

$$N = \begin{cases} \gamma \cdot \left( \frac{46000}{\Delta\sigma_{fict} - 140} \right)^2 & , \Delta\sigma_{fict} > \Delta\sigma_D \\ \gamma \cdot n_0 \cdot \left( \frac{\Delta\sigma_D}{\Delta\sigma_{fict}} \right)^{10} & , \Delta\sigma_{fict} \leq \Delta\sigma_D \end{cases} \quad (7)$$

with  $n_0 = 2 \cdot 10^6$ . Using the fitted median curve above, we find the strength at two million cycles to failure by solving  $N = 2 \cdot 10^6$ , giving the nominal equivalent fatigue strength

$$S_{nom} = 46000 \cdot \sqrt{\frac{\gamma}{2 \cdot 10^6}} + 140 = 196.9 \text{ MPa} \quad (8)$$

which is illustrated in Fig. 5.

#### I. Nominal load

We will here define the equivalent load that is represented by a stress range, repeated two million times, and is damage equivalent to design loads corresponding to the target life. The reliability investigation for the half-scale prototype is made with respect to a target design life of one year. The design loads will be translated into an equivalent fatigue load for the target life.

The damage for a target life of  $T$  years is denoted by  $D_T$  and is calculated using the ANL model without fatigue limit. For the case the equivalent fatigue load range,  $L_{eq}$  (at  $n_0 = 2 \cdot 10^6$  cycles), is below the break point of the ANL curve  $L_{eq} \leq \Delta\sigma_D$ , the damage due to the equivalent fatigue load can be found by

$$D_{eq} = n_0 \cdot \frac{1}{N(L_{eq})} = \frac{1}{\gamma} \cdot \left( \frac{L_{eq}}{\Delta\sigma_D} \right)^{10} \quad (9)$$

For a one-year design load spectrum  $\{(n_i, \Delta\sigma_i)\}_i$ , the  $T$ -year damage is calculated using Palmgren-Miner damage accumulation

$$D_T = T \cdot \sum_i \frac{n_i}{N_i(\Delta\sigma_i)} = T \cdot D_1 \quad (10)$$

where  $D_1$  is the damage for one year. Solving the damage equivalence equation,  $D_{eq} = D_T$ , gives

$$L_{eq,T} = (\gamma \cdot D_T)^{1/10} \cdot \Delta\sigma_D = (T \cdot \gamma \cdot D_1)^{1/10} \cdot \Delta\sigma_D. \quad (11)$$

For a design load spectrum with highest load level below  $\Delta\sigma_D = 172.5$  MPa, the one-year damage becomes

$$D_1 = \sum_i \frac{n_i}{N_i(\Delta\sigma_i)} = \frac{d_1}{\gamma \cdot n_0 \cdot \Delta\sigma_D^{10}} \quad (12)$$

where

$$d_1 = \sum_i n_i \cdot \Delta\sigma_i^{10} \quad (13)$$

is the so-called pseudo damage for one year and the  $T$ -year equivalent fatigue load becomes

$$L_{eq,T} = \left( \frac{T \cdot d_1}{n_0} \right)^{1/10} \quad (14)$$

Note that the reference number of cycles is fixed and does not depend on the time period  $T$ . The design loads are three load levels by means of the property *fictive stress range*. These refer to three different operation cases of the equipment. Since the highest load level  $\Delta\sigma_1 = 120\text{MPa}$  are below the break point  $\Delta\sigma_D = 172.5\text{MPa}$ , the one-year nominal equivalent fatigue load becomes

$$L_{nom} = L_{eq,1} = \left( \frac{d_1}{n_0} \right)^{1/10} = 95.3 \text{ MPa} \quad (15)$$

The nominal values for load and strength gives a nominal safety factor of 1.86. The next step is to investigate the requirement of the safety factor for design, which is based on the VMEA assessment.

#### J. Uncertainty sources identification

The first step in the VMEA analysis phase is to identify the uncertainty sources in load and strength, which is then followed by the size and sensitivity assessment.

##### 1) Strength uncertainties

We identify the following uncertainty sources for the strength

- Scatter (aleatory or physical uncertainty)
- Statistical uncertainty
- Relevance of validation data
- Model error in the fatigue model
- Wall thickness, tolerances
- Wall thickness, corrosion

##### 2) Load uncertainties

The three load levels by means of *fictive stress range*, used as nominal load for strength comparison refer to three different operation cases of the equipment, the two with the highest levels occurring one million times each during the design life of one year. The fictive stress range calculation includes two model assumptions that need to be considered, 1) the thin-walled assumption regarding hoop stress and 2) a chosen stress correction factor for notch influence. Correction factors for temperature and wall thickness are regarded as negligible.

We consider the following uncertainties:

- Correspondence between the given pressures and the pressures generated by the three operations in service.
- Possible model error in the structural analysis.
- Possible error in the chosen notch correction factor.

#### K. Uncertainty size and sensitivity assessment

##### 1) Strength uncertainties

**Scatter and statistical uncertainty.** From the curve fit we found the scatter in terms of the standard deviation  $s = 0.59$  for log-life, and the statistical uncertainty is negligible with such a large sample.

The sensitivity coefficient with respect to the scatter is

$$c = \frac{\ln 197 - \ln 243}{2 \cdot 0.59} = -0.177 \quad (16)$$

where 243 MPa is the stress corresponding to a reduction of logarithmic target life by two standard deviations.

**Relevance of validation data.** The validation data appears to represent different steel qualities used in pressure vessels: "It may also be noted that the database included vessels made from steels that ranged in tensile strength from 370 to 850 N/mm<sup>2</sup>." The uncertainty regarding possible non-relevance to the actual material may then be assumed to be included in the large scatter.

**Model error in the fatigue model.** The model used here with an asymptote at the endurance limit is one of several models in fatigue practice. Other models include the *elementary* Wöhler model neglecting the fatigue limit, and *ordinary* Wöhler curve with other adjustments at the endurance limit. To find a reasonable estimate of the possible model error we here compare with the *elementary* Wöhler curve without endurance limit considerations. We choose to evaluate the model uncertainty at two million cycles, where the fatigue strength of the ANL model is 197 MPa. It turns out that the elementary Wöhler fatigue strength at two million cycles is 151 MPa, and we regard these value as extreme estimates among reasonable models in the region of two million cycles. Assuming a model error uniformly distributed between the ANL model and the elementary Wöhler model gives the standard deviation

$$s_{model} = \frac{\ln(197) - \ln(151)}{\sqrt{12}} = 0.076 \quad (17)$$

**Wall thickness, tolerances and corrosion.** For the wall thickness tolerance and allowed corrosion are both  $\pm 0.5$  mm, giving uncertainty  $s_{wall} = 0.5/\sqrt{3} = 0.289$ . The sensitivity is evaluated to  $c = 0.038$ .

##### 2) Load uncertainties

**Correspondence between the given pressures and the pressures generated by the three operations in service.** These pressures are assessed from numerical analyses and rig tests in laboratory. The real response from sea service is yet not known. We assign an uncertainty of  $\pm 5\%$  here and use the uniform distribution assumption to obtain the standard deviation

$$s_{service} = \frac{0.05}{\sqrt{3}} = 0.029 \quad (18)$$

The sensitivity coefficient is unity in log-scale, since fictive stress is proportional to pressure.

**Possible model error in the structural analysis.** The structural model taken from the reference standard seems

TABLE II  
VMEA TABLE

Input					Result		
Uncertainty components	scatter	uncert.	Sensitivity coefficient c	standard deviation s	Scatter	Uncertainty	Total
<b>Strength</b>							
Strength scatter	x		0.177	0.590	0.104		
Strength model		x	1.000	0.076		0.076	
Wall thickness corrosion	x		0.038	0.289	0.011		
Wall thickness tolerance		x	0.038	0.289		0.011	
<b>Total Strength uncertainty</b>					<b>0.105</b>	<b>0.077</b>	<b>0.130</b>
<b>Load</b>							
Relevance to service		x	1.000	0.029		0.029	
Model error, structural		x	1.000	0.012		0.012	
Model error, Kt		x	1.000	0.078		0.078	
<b>Total Load uncertainty</b>					<b>0.000</b>	<b>0.084</b>	<b>0.084</b>
<b>Total uncertainty</b>					<b>0.105</b>	<b>0.114</b>	<b>0.155</b>

to be based on the thin-walled assumption regarding the hoop stress. The model uncertainty introduced by this model is judged to be  $\pm 2\%$ , which assuming a uniform distribution results in the standard deviation

$$s_{hoop} = \frac{0.02}{\sqrt{3}} = 0.012. \quad (19)$$

The sensitivity coefficient is also here unity in log-scale.

**Possible error in the chosen notch correction factor.**

The notch correction factor is chosen to  $K_t = 1.4$  based on the pressure vessel standard.

However, the radius of the sharpest junction in the piston rod is uncertain. Since the next level in the standard is 1.8, we assume a maximum value in the middle of these two levels, 1.6. Using the uniform distribution, we then assign the uncertainty

$$s_{K_t} = \frac{\ln(1.6) - \ln(1.4)}{\sqrt{3}} = 0.078 \quad (20)$$

with the sensitivity unity.

The uncertainty components are summarised in a spread sheet (Table II) and a pie chart (Fig. 6). The total uncertainty is estimated to 15,5% and is dominated by the strength scatter. Since it includes the uncertainty in the relevance of the used strength specification, which should be possible to reduce by getting relevant data. The other

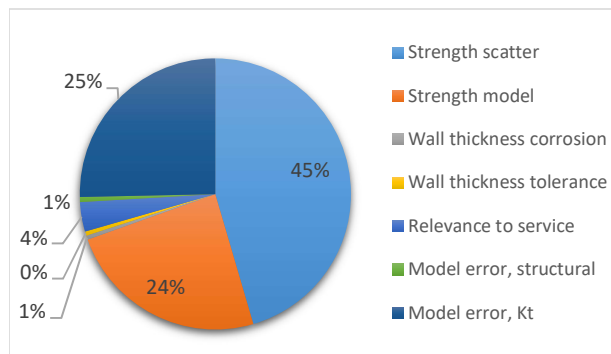


Fig. 6. Pie chart of relative amount of uncertainty contributions.

two important sources are possible model errors in the fatigue model ANL and in the stress concentration factor  $K_f$ . Both these model error uncertainties could probably also be reduced by further investigations.

#### L. Reliability and robustness evaluation

The nominal load and strength, together with the overall uncertainty gives a reliability assessment. The total uncertainty number  $\tau = 0.155$  is multiplied by 1.64 to find the statistical safety distance corresponding to approximately 95% probability of survival,  $1.64 \cdot \tau = 1.64 \cdot 0.155 = 0.25$ .

Reducing the nominal safety distance with this number gives the extra safety distance,  $0.73 - 0.25 = 0.47$ . The corresponding safety factor are the antilog of the distances and the total safety factor is 2.1. The safety factor needed for the statistical part is 1.3, which results in an extra safety factor of 1.6. This result can be regarded as safe, but the margin may not be as large as the damage evaluation by the standard suggests. The results are illustrated in Fig. 7.

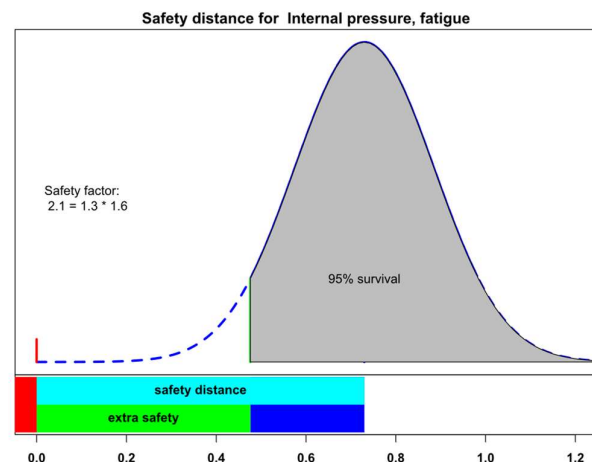


Fig. 7. One-year reliability evaluation for the fatigue case.



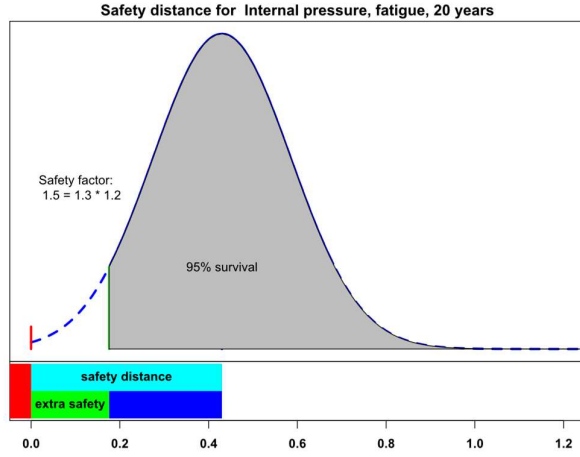


Fig. 8. Twenty-year reliability evaluation for the fatigue case.

#### M. Extension to twenty-year life

It may also be interesting to extend the analysis to a future service situation. We choose to evaluate the reliability for twenty years. The nominal equivalent fatigue strength is defined as the load range at  $n_0 = 2 \cdot 10^6$  cycles and is thus unchanged. However, since the fatigue load accumulates over time, the equivalent fatigue load for 20 years will increase. Recall that the  $T$ -year equivalent fatigue load is calculated as

$$L_{eq,T} = \left( \frac{T \cdot d_1}{n_0} \right)^{1/10} \quad \text{with} \quad d_1 = \sum_i n_i \cdot \Delta \sigma_i^{10} \quad (21)$$

which can be related to the one-year equivalent load as

$$L_{eq,T} = T^{1/10} \cdot L_{eq,1} \quad (22)$$

For  $T = 20$  years the nominal load becomes

$$L_{nom} = L_{eq,20} = 20^{1/10} \cdot L_{eq,1} = 128.6 \text{ MPa} \quad (23)$$

which has increased by 35% compared to the one-year equivalent fatigue load.

The uncertainties in both load and strength do not change, since the uncertainties are modelled in log-scale. In the spread sheet in Table 10 the changes in the analysis are shown. The analysis shows that the extra safety factor is reduced to from 1.6 to 1.2, see Fig. 8.

### VII. CONDITION MONITORING

Monitored signals like temperature, pressure and vibrations are often used as indicators of failure modes, and threshold values can be used for indication of possible failure states or close to failure. Further, deterioration of components can be caused by e.g. wear or fatigue. In this case the loads can be monitored, and an accumulated damage can be computed.

For the pre-tension cylinder, the pressure variations give the accumulated fatigue damage. By using damage models, the remaining safety margin can be calculated and followed over time. This information can then be fed into the maintenance plan, enabling improved preventive maintenance strategies and scheduling.

The fatigue load of the cylinder barrel can be monitored through the internal pressure of the pre-tension module. The rainflow cycles extracted from the pressure signal is used for computing the fatigue damage for a time frame of data. This data processing should be performed on-board the WEC. At regular intervals, say every hour, the pseudo damage value for that period can be transmitted to the on-shore condition monitoring system, where the accumulated fatigue damage is evaluated.

Assume that a prediction horizon of one year is of interest, i.e. the target is that the component shall survive another year. The equivalent load can then be calculated as

$$\tilde{L}_{eq,1}(t) = \left( \frac{d_0(t) + d_1}{n_0} \right)^{1/10} \quad (24)$$

where  $d_0(t)$  is the measured accumulated pseudo damage up to current time point,  $t$ , and  $d_1$  is the expected pseudo damage according to the design loads, see Eq. (21). Thus, by monitoring the pressure the equivalent load can be monitored and the safety distance as function of operating time can be calculated as

$$y(t) = \ln S_{eq} - \ln \tilde{L}_{eq,1}(t) \quad (25)$$

The CorPower C3 WEC has been deployed for wet testing for several months, however the time scale relevant for the monitoring case would be in the order of years. Therefore, the fictive example in Fig. 8. is used for demonstration and does not correspond to actual measured data. The black lines in Fig. 8 represent the design loads and the blue thick lines the monitored loads. Further, the dashed lines show the 50% reliability curves,  $y(t)$ , while the solid lines show the 95% reliability curves,  $y(t) - 1.64\tau$ , and the failure limit of zero is shown as a red line. Note that Fig. 7 represents time zero in Fig. 8. We can observe that, in this fictive example, the monitored load is more damaging than the design loads. However, it can be concluded that the cylinder barrel is still safe for operation.

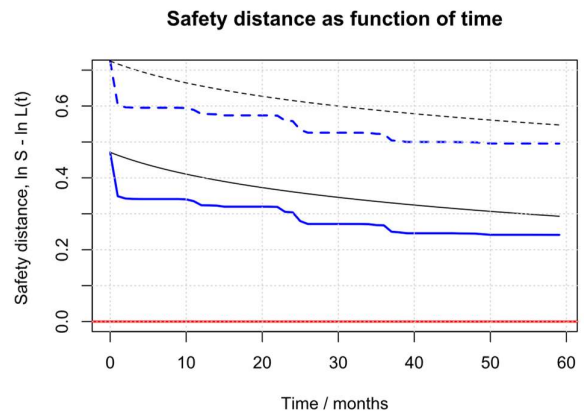


Fig. 9. Safety distance for one-year prediction horizon.

## VIII. DISCUSSION

For the fatigue strength example, the reliability methodology used by the pressure vessel standard is based on demands on the calculated damage. The margin given by this demand should include all uncertainty sources including possible model error. However, it may be doubtful if this standardised margin is valid for cases in the vicinity of the fatigue limit, since the possible model error here is substantial. The VMEA methodology, by choosing margins on the load dimension, gives a more robust judgement about model uncertainty and by comparing with other models its possible size can also be assessed.

The standard gives good guidance for performing design calculations, however it does not give any support on how new knowledge or data can be used in order to reduce the required safety margins or to guide improvement actions. On the contrary, for VMEA this kind of new information is directly reflected in the uncertainty numbers, and thus has a direct impact on the statistical safety margin. However, our recommendation is to complement the statistical safety factor by an extra safety factor not based on statistics but on economical and safety considerations. For both VMEA examples, the comparison of different uncertainty causes is a good basis for identifying possible new investigations or improvements actions that can give better knowledge on the uncertainties and hopefully result in a decrease of the overall uncertainty.

The VMEA methodology can also be a good basis for condition monitoring, where the accumulated equivalent fatigue load can be monitored, and the uncertainties may be updated based on operational data. This gives the possibility to predict the remaining life and its uncertainty, which can be valuable input to the maintenance planning.

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