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Hydrostatic Stability of Floating Oscillating Water Columns

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Fig. 1. Wandoo Concrete Gravity Structure during tow–out of a casting basin in Western Australia [1].

Abstract—**Oscillating water columns (OWC) that are integrated to existing structures (e.g. breakwaters) or fixed to seabed have been shown to be effective wave energy conversion systems. Floating OWC have recently been studied from the viewpoint of design optimization and increasing efficiency. The floating variants provide the ability to deploy OWC systems in varying water depths, and can improve versatility while maintaining minimum disturbance to the sea floor. The air chamber within a floating OWC is similar in many ways to an air cushion used in offshore applications. For the case of a concrete gravity sub-structure used for oil and gas production, air cushions are located within skirted base compartments to artificially lighten the structure during tow-out. The air cushion is sealed by a "water plug" so as to prevent air egress into the open water. The air chambers affect the overall hydrostatic stability in many ways, and can even provide favorable consequences at different length scales. In this paper, we use first principles to obtain the restoring moments for a generic floating oscillating water column system with multiple compartments. A control volume approach is used where the water masses are treated as added weights. The working fluid (air) is treated using the adiabatic law and a simple power take off system is included. The resulting formulations are closed form expressions for the restoring moments in roll and pitch of the structure, which in turn are cast into stability metrics like the metacentric height. Results are discussed in terms of the extraction efficiency of the built in OWC.**

Keywords—**air cushion support, oscillating water columns, hydrostatic stability, floating OWC.**

I. Introduction

THIS author was involved in a series of studies to assess the viability of construction of concrete structures for offshore oil and gas production in Western Australia [1]. The second-generation concrete platforms used in the industry comprise of large box type base structure (Fig. 1). The base structure can often comprise of skirted compartments for foundation support (Fig. 2). During tow-out in shallow water conditions, these compartments can enclose a cushion of air thus artificially lightening the structure. The air is released once deeper water depths are reached. The air is sealed during the tow-out process by the so-called water plug (Fig. 2). The stability of a typical compartmented concrete sub-structure during tow-out was studied by a number of references, see e.g. [2 - 4]. In particular, [4] examined the stability of a freely floating air cushion structure without the influence of external forces. Closed form expression for stability metrics were developed and validated against experimental data. It was shown that air compressibility plays a profound role in assessing the stability of prototype structures. The results showed that a compartmented structure supported on air cushions can have stability comparable to a closed box type of similar geometry. This counter-intuitive result was explained as being due to the water plug being entrapped within the structure and hence contributes to lowering the center of gravity and increasing stability.

The idea of applying compartmented concrete structures for floating oscillating water columns (OWC) is appealing. As one of the earliest developed wave energy conversion concept, the OWC has been studied quite extensively [e.g. 5]. Floating variants of OWC can be attractive because of their applicability to different water depths, while maintaining minimum disturbance to the sea floor. If concrete OWC are shown to be viable, the structure can be fabricated in suitably modified port and harbour areas, thus generating local employment. Such structures can be individual rafts, or combined together

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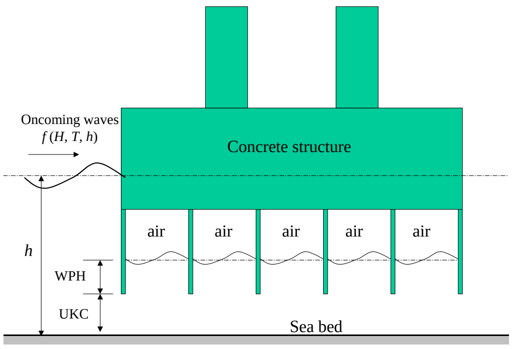

Fig. 2. Air cushion supported concrete gravity structure (CGS) during tow-out in shallow water [3]; (WPH = water plug height, UKC = under keel clearance).

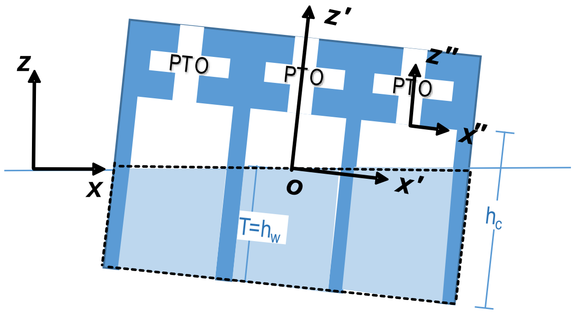

Fig. 3. A generic compartmented structure supporting multiple oscillating water columns, with each OWC connected to a power take off (PTO) system. The draft T is assumed equal to the water plug height (h_w). Compartment height is h_c.

to form a train of rafts. The concept could also be integrated into floating breakwaters, and can supplement similar ideas explored by e.g. [6 - 7].

The hydrostatic stability of compartmented structures used as floating oscillating water columns needs evaluation, since the presence of an energy generating turbine in each compartment that connects the compartment to the atmosphere can change the nature of the air chamber, when compared with ones shown in Fig. 2. In this paper, we present an approach developed from first principles for obtaining the stability metrics of a floating OWC structure. Such formulations can be widely applicable for similar structures with various types of power take off mechanisms.

II. Mathematical Formulation

The formulation presented here closely follows a similar one developed for air cushion supported offshore platforms in [4]. The generic platform considered here is a concrete box structure of length L, width B and height H whose side view is shown in Fig. 3. The box has a number of compartments of cross-section l x b, which are open to sea. Each of these compartments can be connected to a power take off system so that they can operate as an oscillating water column, and can harness energy from oncoming waves. Variants to the geometry as well as more efficient power take off arrangements (such as shared turbines across compartments) may be considered in the future, but for this paper a simple geometry will suffice.

Three coordinate systems are defined as below:

- Global – (x, y, z)
- Body fixed – (x', y', z')
- Compartment fixed – (x'', y'', z'')

The body can have motions in six degrees of freedom ζ_i, comprising of three translations $(i = 1, \cdots 3)$, and three rotations $(i = 4, \cdots 6)$. The origins of the global and body fixed coordinates are coincident at initial time, and located at the intersection of the planes of symmetry and the water plane. After some time, translational motions can displace the origin of the body fixed coordinate system from the global origin. In still water, the body floats freely at draft T due to equilibrium between the forces of weight and buoyancy. To assess its hydrostatic stability, the body is given an arbitrarily small rotational displacement ζ_5 about the y axis. The goal of this paper is to develop formulations for the restoring moments on the body in response to this displacement, when multiple OWC's are present in the body.

The position vector of any point within a compartment is given by

$$\mathbb{r} = \mathbb{r}_0 + \mathbb{r}' + \mathbb{w} \times \mathbb{r}' \tag{1}$$

$$\mathbb{r}' = \mathbb{r}'_{ij} + \mathbb{r}'' \tag{2}$$

where double struck symbols denote vector quantities. Subscript 0 denotes the origin of the body fixed system whose coordinates are $(\zeta_1, \zeta_2, \zeta_3)$ as mentioned before, and subscript ij denotes the origin of the coordinate system fixed to the compartment with indices (i, j). The rotation vector is denoted by $\mathbb{w} = (\zeta_4, \zeta_5, \zeta_6)$.

We consider a control volume denoted by a dashed line shown in Fig. 3. The restoring moment on the body arises due to an imbalance between the buoyancy and weight in the tilted condition. The mass enclosed within the control volume comprises of the mass of the body (m_s) and those of the entrapped (m_{ij}) water within each compartment. Thus

$$m = m_s + \sum_i \sum_j m_{ij} \tag{3}$$

The internal free surface is horizontal in the global coordinate system. With respect to the compartment fixed system, the surface rotates about the midpoint of the surface. Thus the mass of water within a compartment is given by

$$m_{ij} = \rho lb\left(h_c + z''_{c\_ij}\right) \tag{4}$$

where z''_{c_ij} is the vertical coordinate of the midpoint of the internal water surface in the compartment fixed coordinate system. This height is determined by the pressure within the compartment, which in turn is decided by the PTO and its operation. Once the mass is known, the moment due to weight in global coordinates is given by

$$\mathbb{M}_G = \mathbb{r}_G \times (0, 0, -mg) \tag{5}$$

The position vector of the center of gravity is more easily found in the body fixed coordinates. Thus, we get by using (1)

$$\mathbb{r}_G = (x'_G + \zeta_5 z'_G, y'_G, z'_G - \zeta_5 x'_G) \tag{6}$$

The y-component of the moment is of great interest. This is evaluated to be

$$M_{Gy} = mg(x'_G + \zeta_5 z'_G) \tag{7}$$

The hydrostatic restoring moment is obtained by integrating the hydrostatic pressure acting on the control volume, and is given by

$$\mathbb{M}_B = -\rho g \iint_S z\,(\mathbb{r} \times \mathbb{n}) dS \tag{8}$$

Here S is the wetted surface of the control volume and $\mathbb{n}$ is the outward normal of the fluid domain at every point on S. Evaluation of this integral is straightforward and well described in references (e.g. [8]). The component that acts about the y-axis comprises of two elements, one that is due to the vertical position of the center of buoyancy (z'_B) and the other due to the second moment of area of the water plane (S_{yy}). Thus

$$M_{By} = -\rho g\big(V z'_B + S_{yy}\big)\zeta_5 \tag{9}$$

Equations (7) and (9) together give the net restoring moment on the body due to a rotation about the y-axis. This can be written in terms of a restoring coefficient as

$$M_{By} - M_{Gy} = -C_{55}\zeta_5 \tag{10}$$

Class rules commonly use the term "metacentric height" as a metric for platform stability. This can be obtained by dividing the restoring coefficient C_{55} by the weight of displaced water. For a closed box, we can let $z'_B = -T/2$, and the second moment of area of the water plane $S_{yy} = {L^3 B}/{12}$. Then the metacentric height of a simple box shaped structure is [9]

$$GM_{box} = \frac{L^2}{12T} - \frac{T}{2} - z'_G \tag{11}$$

The requirement for positive stability is that the metacentric height be greater than zero.

In conventional naval architecture, the displaced volume is that due to the structure alone, devoid of compartments. Here we consider two conditions unique to our application:

- Turbine blocked completely, i.e. the air mass is entrapped within a compartment, and
- Turbine in operation, and a pressure drop is applied across the turbine.

A. Case 1: Turbine blocked

In this case, we consider all the turbines to be blocked and the air masses within the compartments are entrapped. Considering a single compartment, the volume of air in the tilted condition follows from (4) as

$$v_{ij} = -lb z''_{c\_ij} \tag{12}$$

where the minus sign is a consequence of the choice of the orientation of the z axis in the compartment fixed coordinate system. Invoking the linear adiabatic law, the incremental change in air volume within the compartment is

$$\Delta v_{ij} = -\frac{1}{\gamma}\frac{v_0}{p_0}\Delta p_{ij} \tag{13}$$

where the initial conditions common to all compartments are the volume $v_0 = lb\,(h_c - h_w)$ and pressure $p_0 = p_{atm} + \rho g\,(T - h_w)$, and γ is the polytrophic constant. The change in pressure is then related to the change in the vertical coordinate of the internal surface in the global coordinate system. This then gives an explicit expression for the quantity z''_{c_ij} needed to evaluate the water masses, as

$$-z''_{c_{ij}} = (h_c - h_w)\left(1 - \frac{x'_{ij}\,\zeta_5}{\gamma p_0/\rho g + (h_c - h_w)}\right) \tag{14}$$

This quantity is a function of the compressibility of the air cushion, as well as the distance of the compartment from the origin. Substituting into (4), the water mass is evaluated, and that then enables evaluating all the terms in (7) that depend on ζ_5. When evaluating terms in (7) the components of mass and their individual centers of gravity should be considered. For example,

$$m x'_G = m_s x'_{s\_G} + \sum_i \sum_j m_{ij}\,(x'_{ij} + x''_{ij\_G}) \tag{15}$$

where x'_{s_G} and x''_{ij_G} are the x-coordinates of the centers of gravity of the body and the water masses respectively in the corresponding coordinate system. Upon performing the algebra, and following (10), the expression obtained for the restoring coefficient in pitch is shown in (16). This equation reveals several interesting features. The first term is the combined effect of buoyancy. The second term is the moment due to gravity of the body mass, while the third term is the effect of the weight of the entrapped water. Since this is closer to the keel of the box, the effect is to lower the net center of mass and hence improve the stability. The fourth term is the destabilizing effect of internal water surfaces (N – total number of compartments). The last term is dependent on the pressure of the air chamber, and is associated with the compressibility of the air. This term increases the destabilizing effect of the internal water surfaces.

$$\begin{aligned} C_{55} = \; & \rho g\left(V z_B' + S_{yy}\right) - m_s g z_{s_G}' \\ & + \sum_i \sum_j m_{ij}\, g\left(\frac{h_w}{2} - \mathrm{T}\right) - \rho g \frac{l^3 b}{12} N \\ & - \frac{\rho g l b (h_c - h_w)}{\gamma p_0 / \rho g + (h_c - h_w)} \sum_i \sum_j {x_{ij}'}^2 \end{aligned} \tag{16}$$

To find the metacentric height, we choose the displaced volume to be equivalent to the control volume, so as to obtain a direct comparison with the metacentric height of an equivalent box. Thus we could define the displaced weight as $\rho g V$. Letting the metacentric height as GM, we get from (16)

$$\begin{aligned} GM = \; & \left(z_B' + \frac{S_{yy}}{V}\right) - \frac{m_s}{\rho V} z_{s_G}' \\ & + \frac{1}{\rho V} \sum_i \sum_j m_{ij}\left(\frac{h_w}{2} - \mathrm{T}\right) - \frac{l^3 b}{12 V} N \\ & - \frac{l b (h_c - h_w)}{V\left(\gamma p_0 / \rho g + (h_c - h_w)\right)} \sum_i \sum_j {x_{ij}'}^2 \end{aligned} \tag{17}$$

The requirement that the metacentric height remain positive then requires that the center of gravity of the structure must then satisfy the condition

$$\begin{aligned} \frac{m_s}{\rho V} z_{s_G}' < \; & \left(z_B' + \frac{S_{yy}}{V}\right) \\ & + \frac{1}{\rho V} \sum_i \sum_j m_{ij}\left(\frac{h_w}{2} - \mathrm{T}\right) - \frac{l^3 b}{12 V} N \\ & - \frac{l b (h_c - h_w)}{V\left(\gamma p_0 / \rho g + (h_c - h_w)\right)} \sum_i \sum_j {x_{ij}'}^2 \end{aligned} \tag{18}$$

B. Case 2: Turbine active

The attachment of a power take off system to an air chamber may be equated to a softer "air spring" from a stability point of view. The pressure within an air chamber is altered by the presence of a PTO. If the PTO is a power generating turbine, then the pressure drop across the turbine combined with the atmospheric pressure decides on the pressure within a compartment. This pressure drop is quite different between the "inhale" and "exhale" phases of operation, and is well described by several models, see e.g. [10]. For simplicity, the difference is ignored here, and the two phases of operation are considered similar. The modification of the air chamber stiffness due to a PTO is incorporated into our formulation by an equivalent change in the polytrophic constant γ. This idea has its roots in the artificial compressibility concept used in computational approaches like Smoothed Particle Hydrodynamics e.g. [11]. The idea is also used in studies, e.g. [5], that accommodated the thermodynamic effects of turbine operation into OWC air chamber compressibility.

It is fairly straightforward to show that the stiffness of an enclosed air chamber is directly proportional to the negative rate of change of pressure with respect to volume. Rewriting (13), we get

$$-\frac{dp_{ij}}{dv_{ij}} = \frac{\gamma p_0}{v_0} \tag{19}$$

In the presence of a PTO system, this gets modified due to the air volume flow rate (Q_t) through the turbine. Following the linearized relationship between Q_t and Q developed in [12], we may write

$$-\frac{dp_{ij}}{dv_{ij}} = \frac{\gamma p_0}{v_0}\left(1 - \frac{Q_t}{Q}\right) \tag{20}$$

Comparing with the form of (19), we propose that the equivalent polytrophic constant of the air chamber with a PTO is

$$\gamma_t = \gamma\left(1 - \frac{Q_t}{Q}\right) \tag{21}$$

Since the power output is proportional to the flow rate, we can equate the ratio of the flow rates to the ratio of power outputs across the turbine and that across the internal water surface. This then translates to the ratio of the overall power conversion efficiency to the hydrodynamic efficiency of an OWC. If this ratio is denoted as η_t, then

$$\gamma_t = \gamma(1 - \eta_t) \tag{21}$$

By replacing γ with γ_t in (16) – (18), equivalent stability metric for a floating OWC system is obtained.

III. Results and Discussion

The restoring coefficient given in (16) was applied to a compartmented box of dimensions 0.5 m x 0.5 m and found to agree well with experimental data, see [4]. Furthermore, it was also found to accurately predict the marginal stability of a single compartment box, where established formulations suggest that the metacentric height should be negative due to a large internal free surface.

TABLE I
PROTOTYPE FLOATING OWC PARTICULARS

Symbol	Quantity	Value
L	Length	50 m
B	Breadth	50 m
T	Draft	Variable from 3.5 – 9.5 m
KG	Structure vertical center of gravity	Variable from 3 – 15 m
N	Total no. of compartments	9 (3 x 3)
h_c	Compartment height	10 m
l	Compartment length	15.4 m
b	Compartment width	15.4 m
h_w	Water plug height	Variable, equal to draft.

For the purposes of this paper, we consider a prototype of dimensions given in Table 1. The square shaped body is assumed to have nine compartments arranged in a honeycomb pattern. The height of the air chamber is assumed to be larger than the maximum draft that will be studied. An equivalent closed box is also considered of similar drafts to obtain a baseline for comparison.

The air chambers are assumed to be at atmospheric pressure initially, i.e. $h_w = T$. It is to be noted that the structural mass will need to be modified for each draft condition, as per the following

$$m_s = \rho T \left(LB - \sum_i \sum_j lb \right) \tag{22}$$

Figure (4) shows a comparison between the metacentric heights of the closed box and the compartmented structure in Case A condition, i.e. with all turbines blocked. The center of gravity is fixed for the purposes of this figure at 10 m from the keel, in line with the top of the compartment. The results show that the curves follow similar trends with the compartmented structure values being slightly lower until a draft value of 7.5 m.

In general the GM values are quite large as is typical for a flat bottom structure like a closed box. Such structures are considered quite "rigid" in terms of their pitching response. For the case A configuration, we can examine the change in GM value as a result of changes in the center of gravity of the structure and as a result of changes in the draft, which is obtained by changing the structure mass itself. This variation is shown as a contour plot in Figure 5. The GM decreases as the height of the center of gravity increases, with a more gradual dependence on the draft.

For Case B configuration, in order to study the effect of the efficiency ratio, we again set a fixed value of the center of gravity at 10 m from the keel, and study two different draft conditions: a low value of 4.5 m, and a higher value of 8.5 m. Figure 6 shows the results. As the efficiency ratio increases and more energy is extracted, the metacentric height decreases quite dramatically. The decrease is modest for ratios of up to 0.5, and then decreases more steeply at the higher values of efficiency ratio. One may argue that efficiency values higher than 0.5 are harder to attain in reality. Thus GM reduction in the order of 20 – 30% may be reasonable to expect during the operation of the OWC.

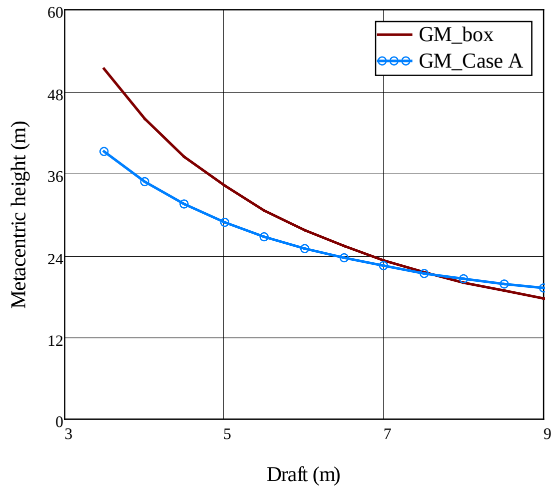

Fig. 4. The metacentric height of a closed box compared to a compartmented box in Case A configuration. The abscissa corresponds to the draft of the structures. The center of gravity is fixed at the top of the compartment (10 m).

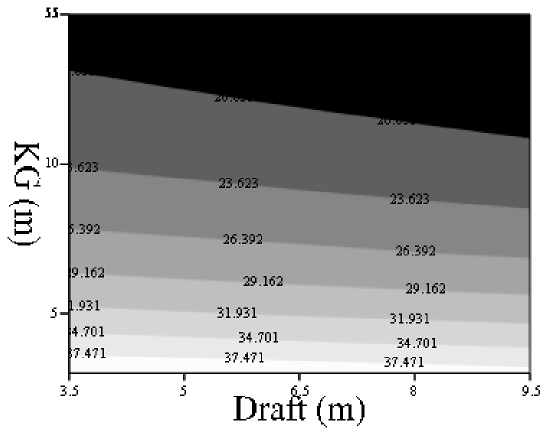

Fig. 5. Contour plots of the metacentric height of a compartmented box in Case A configuration. The abscissa corresponds to the draft of the structure, while the ordinate corresponds to the height of the center of gravity from the keel of the structure.

IV. CONCLUSIONS

We here considered a generic compartmented structure supporting multiple oscillating water columns and examined the former's floating stability using a fundamental approach. We found that the stability depends strongly on the energy extraction efficiency of the OWC, and in turn is an operating parameter. One could expect 30 – 40% reduction in the metacentric height

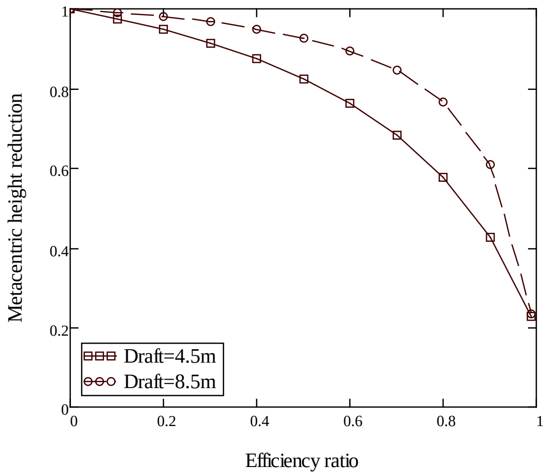

Fig. 6. The metacentric height reduction of the floating OWC as a function of the ratio of overall power conversion efficiency to the hydrodynamic efficiency. Two different drafts (4.5 m and 8.5 m) are shown. Vertical center of gravity is kept constant at 10 m.

due to OWC operating with an efficiency ratio of 0.5. The implications of these findings may be relevant beyond floating stability considerations. The natural frequencies of the structure in roll or pitch depend on the metacentric height in the corresponding planes. Thus a 40% reduction in metacentric height could result in a 30% increase in the natural frequency, provided the added moment of inertia is approximately the same. Further work on this could involve a set of experiments on a structure with multiple columns to ascertain the natural frequencies.

This paper is a first attempt at developing stability metrics for floating OWC systems. The formulations developed here show considerable promise for practical design and development applications. Metacentric height formulae could be used in sizing calculations to quickly screen novel configurations prior to detailed numerical simulations and experimental studies.

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