Proceedings of the 13th European Wave and Tidal Energy Conference 1-6 Sept 2019, Naples, Italy

Optimization of shape and control of nonlinear Wave Energy Converters

Jiajun Song, Ossama Abdelkhalik, Shadi Darani

Abstract—**This paper presents an optimization approach to design axisymmetric wave energy converters (WECs) based on a nonlinear hydrodynamic model. This paper adopts analytical formulas for the nonlinear Froude-Krylov force of complex buoy shapes, by combining basic shapes in an optimal sense. The time domain Froude-Krylov force can be computed for a complex buoy shape, then is decomposed into its static and dynamic components. The excitation force and the hydrostatic force are calculated by the dynamic and static component of the nonlinear Froude-Krylov force. A nonlinear control is assumed in the form of nonlinear and linear damping terms. Genetic Algorithms are implemented in searching for the optimal buoy shape along with the optimal control coefficients simultaneously. Simulation results presented in this paper show that it is possible to find non traditional buoy shapes that leverage nonlinear hydrodynamics to harvest more energy.**

Index Terms—**Wave Energy Conversion, Nonlinear WEC, Nonlinear Optimization, Nonlinear Froude-Krylov Force.**

I. Introduction

WAVES can be a reliable source of renewable energy if wave energy converters (WEC) can be operated in an economic way under various sea conditions. Once a location of WEC device is identified, the shape of the buoy and the control can be optimized to maximize the harvested energy. Such optimization can be achieved through the time domain simulation of the WEC device under the statistic wave climate data. Alternatively, a specific shape of buoy and a specific control can be obtained through optimization using the statistic wave spectrum data of the fixed location as the input. Shape optimization of WECs based on linear model was carried out by Korde and Jiajun [1]. Limited cases of shape designs were tested, preliminary evaluations were on the exciting force and the radiation damping. Better energy conversion was achieved from non-cylindrical shapes of the buoy.

The dynamic modeling of WEC has been a topic for marine energy study for a long time. According to Falnes [2], a linear dynamic model is accurate enough to study the energy extraction and the body motion in a small wave amplitude environment. However, the linear assumption is not sufficient to deal with a realistic wave spectrum environment or a large motion of a buoy due to high wave amplitudes. A nonlinear hydrodynamic model of WEC device can result in significant improvement in realistic time domain simulation of wave energy extraction.

The research about the extraction of weave energy starts from mid of 1970s by Budal [3] and Salter [4]. In recent years there have seen a large amount of applications, as reviewed in Falnes's [5] and Ringwood's review paper [6]. The typical linear dynamic equation of motion for a floater of mass m can be described as shown in (1) according to Falnes's book [2], Ringwood and Korde's control theory [7].

$$m\ddot{x} = f_e + f_r + f_s + u \tag{1}$$

Where, $\ddot{x}$ is the acceleration of the buoy. f_e is the excitation force given from the incoming wave around the immersed surface. For a spectrum of incoming waves with n different frequencies, each frequency components of the wave spectrum contains the wave amplitude A_i and the wave frequency ω_i. The excitation force coefficient F_{ei} and the phase shift from incoming wave to excitation froce ϕ_i, where F_{ei} and ϕ_i can be calculated using Nemoh from the input of a wetted surface mesh. The total time domain excitation force f_e at time t can be expressed by the Fourier series of the frequency components as shown in (2).

$$f_e(t) = \sum_{1}^{n} F_{ei} A_i \cos(\omega_i t + \phi_i) \tag{2}$$

The radiation force, f_r, is due to the radiated wave from motion of the floater. f_r can be computed from the radiation force term h_r as shown in (3).

$$f_r(t) = \int_0^{\infty} h_r(\tau)\dot{x}(t-\tau)d\tau \tag{3}$$

The hydrostatic force f_s represent the spring-like characteristic of the interaction motion between floater and water as shown in (4), where k is the hydrostatic stiffness due to buoyancy.

$$f_s(t) = kx \tag{4}$$

And u is the external power-take-off force or the control force. Hydrodynamic coefficients in (2) (3) and (4) such as F_{ei}, ϕ_i, h_r and k are calculated from mathematical models of WECs.

Mathematical models for WECs are an essential tool for device design, optimization, control and management. The choice of an appropriate model depends on the specific requirements demanded by the intended research project [8]. In particular, for the design of a

ID: 1700, Track: WDD

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WEC device working in a realistic ocean wave environment with extreme events and load studies, a nonlinear hydrodynamic model is required to simulate the motion and the energy extraction with expected accuracy [9].

One of the mathematical models of non-linear WEC problems is the CFD approach, which uses continues meshing method [10]. Similar simulations have been tested by Penalba and Ringwood [11]. This CFD approach requires large amount of computation time, because a new mesh is needed for every update of the propagation step. A computational hydrodynamic model method was chosen to be implemented in this paper.

Giorgi and Ringwood [12] came up with a simplified format of nonlinear hydrodynamic model:

$$m\ddot{\xi} = F_{FK_{st}} + (F_{FK_{dy}} + F_D) + F_R + F_{PTO} \qquad (5)$$

This analytical solution of the nonlinear FK force was developed by Ringwood [13] for WECs of simple geometries. Compare (1) with (5), terms in the linear model can be replaced with non-linear term from the Froude-Krylov force: $f_s = F_{FK_{st}}$, $f_e = F_{FK_{dy}} + F_D$. The hydrodynamic model can be expressed as (5). Where $F_{FK_{st}}$ is the static Froude-Krylov force, given as the difference between the gravity force and the Archimed force, $F_{FK_{dy}}$ is the dynamic Froude-Krylov force, F_D is the diffraction force from the undisturbed incoming wave. F_{PTO} is the force of the power-take-off (PTO) device, or the control force. Airys wave theory for deep water waves is used to compute the pressure along the submerged surface. The integration of the pressure along the submerged surface contributes to the nonlinear Froude-Krylov force calculation.

Regrading the radiation force term $f_r = F_R$. Falnes [2], Clement and Ferrant [14] showed that the nonlinearities of radiation and diffraction force are assumed to be negligible when the device dimension is considerably smaller than the wave length. This assumption holds true as the dimension of the buoy discussed in this paper is less then 1/5 of the wave length. Meriguad et al. [15] showed that the response of a heaving point absorber is mainly affected by the nonlinear FK forces, while the nonlinear radiation and diffraction force have minor effects on system dynamics. Validation of a nonlinear Fourde-Krylov model with linear radiation and diffraction term was tested using a real wave tank by Gilloteaux [16] and Guerinel et al. [15]. The nonlinear model shows a significant improvement of accuracy with respect to a full-linear model and good agreement with experimental measurements. Similar results were obtained by Giorgi and Ringwood [17].

This paper presents a Genetic Algorithm implemented optimization approach to design axisymmetric WECs based on a nonlinear hydrodynamic model. The nonlinear hydrodynamic model gain non-linearity from the nonlinear Froude-Krylov force. The proposed optimization tool provides the optimal solution of the buoy shape and the optimal control simultaneously. The optimal solutions in proposed paper mainly focus

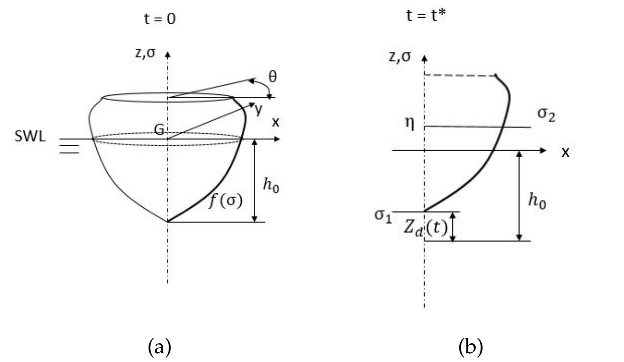


Fig. 1. The surface of an axisymmetric heaving device with generic profile $f(\sigma)$. 1a shows the equilibrium position at the still water level (SWL) and the draft h_0; 1b shows the free elevation η and the device displacement z_d after a time t^*. The pressure is integrated over the surface between σ_1 and σ_2.

on energy conversion, further investigation on construction cost of WEC device will be considered in future research.

II. Optimization of the Buoy Shape

There are several energy extraction concepts which can be categorized into three categories based on the interaction between device and ocean wave [2]. First, the oscillating body design [18]. Second, the oscillating water column design [19]. And third, the over-topping converters [5], [20].

This paper focus on the oscillating body design, specifically, the axisymmetreic oscillating body design. Reasons of selecting axisymmetric body design are: only one direction of the incoming exciting wave is needed to be considered, convenience of the computation of an analytical solution for the nonlinear Froude-Krylov force.

Giorgi developed a format to describe an axisymmetric geometry with a fixed vertical axis as in (6) [21].

As shown in Fig. 1, the surface of an axisymmetric body can be described in parametric cylindrical coordinates:

$$\begin{aligned} x(\sigma,\theta) &= f(\sigma)cos\theta \\ y(\sigma,\theta) &= f(\sigma)sin\theta \\ z(\sigma,\theta) &= \sigma \\ \theta \in [0,2\pi) &\cap \sigma \in [\sigma_1,\sigma_2] \end{aligned} \qquad (6)$$

Based on the superposition of integral, the total vertical FK force acting on a surface S can be decomposed into smaller forces acting on corresponding areas in (7).

$$\begin{aligned} F_{FK_z} &= \iint_S P\vec{n}dS \\ &= \int_0^{2\pi}\int_{\sigma_1}^{\sigma_2} Pf'(\sigma)f(\sigma)d\sigma d\theta \\ &= \int_0^{2\pi}\left[\sum_{i=1}^{N-1}\int_{\hat{\sigma}_i}^{\hat{\sigma}_{i+1}} Pf'(\sigma)f(\sigma)d\sigma\right]d\theta \\ &= \sum_{i=1}^{N-1}\int_0^{2\pi}\int_{\hat{\sigma}_i}^{\hat{\sigma}_{i+1}} Pf'(\sigma)f(\sigma)d\sigma d\theta \end{aligned} \qquad (7)$$

where $\hat{\sigma}_1 = \sigma_1, \hat{\sigma}_N = \sigma_2$

Complex WEC shapes constructed by several simple shape elements were tested in the nonlinear FK model. Decomposition of the whole shape gives several sections, each section S_i can be described by just two variables α_i and h_i, as shown in Fig. 2

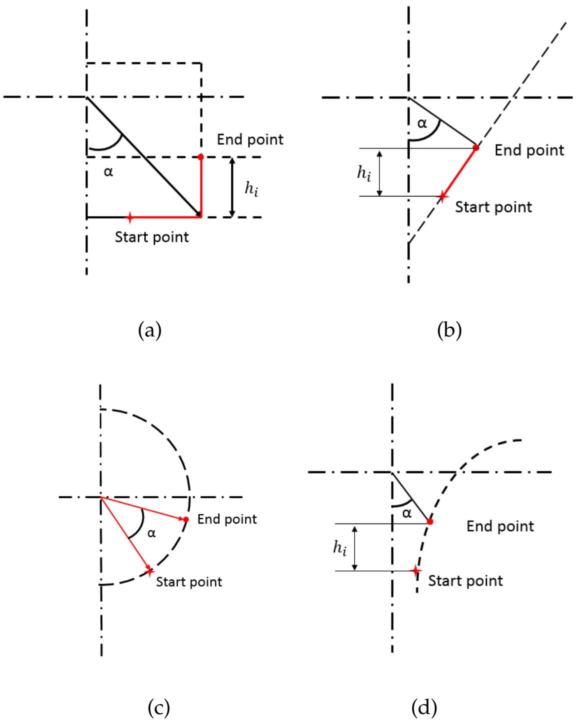


Fig. 2. Each section i of decomposed shape can be described by two variables α_i and h_i or less.

To build the outline for the mesh of the buoy, the bottom point of the shape is used as the start point of the outline. Two design variables α_i and h_i will define the coordinate of the end point for each section. With the end point defined and the start point inherited from the previous section, coordinates of new section can be defined depends on the section type.

The optimization method of Genetic Algorithm (GA) was selected to carry out the optimization procedure, due to the stochastic choice of elements to construct the total shape of WEC [22]. Size of the design variables for each section was reduced to 2 to improve the efficiency of GA.

In standard Genetic Algorithms, the variables of the optimization problem are coded in chromosomes. Each chromosome represents a solution and consists of the variables that are coded as genes. The objective of optimization determines the fitness of the solution.

In this paper, a chromosomes is defined in the following format (8):

$$X_i = [N_i, S_{ty1}, ...S_{tyN}, X_1(1,2), ...X_N(1,2)] \tag{8}$$

Definition of each element in the chromosome is: N_i, the number of active sections. Meaning how many sections will contribute to the total shape. $N_i \in [1, N]$ where N is the maximum number of sections, and N_i is an integer.

S_{tyi} is the geometry type of the i_{th} section. $S_{tyi} = 1$ is a cylindrical shape, $S_{tyi} = 2$ is an oblique line, $S_{tyi} = 3$ is an arc of circumference, and $S_{tyi} = 4$ is an exponential profile.

$X_i(1,2)$ are the design variables of each section, $X_i(1) = \alpha_i, X_i(2) = h_i$. Where $\alpha_i \in (0, 90°)$, $h_i \in (0, \inf)$ and defined in Fig. 2.

III. Optimization of the Control

The dynamic model of WECs may have nonlinearities due to several reasons such as nonuniform buoy shapes and/or nonlinear power takeoff units. The nonlinear effects also arise from wave-buoy interactions and nonlinear incoming waves. The nonlinear control algorithm is needed for such nonlinear WEC designs to optimize the energy conversion [23].

A nonlinear controller was developed in Michigan Technological University (MTU) by Abdelkhalik and Darani [22]. Control force was constructed as (9):

$$f_c = \sum_{i=1}^{N} a_i z_i + \sum_{j=1}^{M} b_j |\dot{z}^j| sign(\dot{z}) \tag{9}$$

Where f_c is the nonlinear control force, a_i and b_j are the constant control coefficients, N and M are the number of nonlinear terms that determine the order of the nonlinear force. This control algorithm shows improvement in the energy extraction compared to the traditional linear resistive loading control method [24].

IV. Numerical Results

A preliminary research was conducted on a 3-section complex WEC shape design. Optimal solutions of shape designs with respect to different cost functions are present below. Boundary element solver Nemoh was used to compute the hydrodynamic coefficients of the nonlinear shape and of the base line cylindrical buoy. Wave spectrum with the peak period $T_p = 8s$ and the significant wave height $H_s = 0.8m$ was used in the simulations. The nonlinear controller (10) was chosen. And All optimized shape cases were compared with a reference cylindrical WEC with the similar mass of each WEC designs, traditional resistive control was applied to the reference cylindrical buoy. The traditional resistive control coefficients were optimized such that the energy extraction is efficient as the comparison baseline.

$$u = F_{PTO} = b_1\dot{x} + b_2\dot{x}^3 \tag{10}$$

The energy converted by the WEC device over a period of time T can be formulated as:

$$E = \int_0^T P(t)dt = -\int_0^T F_{PTO}\dot{z}dt = -\int_0^T u\dot{z}dt \tag{11}$$

The cost functions of each test were customized to have specify weight on different aspects of the criteria for a better WEC design. Such cost functions were designed to emphasize on average power extraction, or to emphasize on maximum energy conversion per unit mass. The optimization were simulated on a realistic Bretschneider spectrum, with a significant wave height of $H_s = 0.8$ m and peak period of $T_p = 8$ seconds. The Bretschneider spectrum wave is sampled at 34 frequencies for simulation, as shown in Fig. 3. Only one wave spectrum was used in this paper to compare different buoy-shape designs that emphasizing on different energy conversion criteria. Change in the wave spectrum would leads to change in the optimal buoy shape and change in theoptimal control coefficients.

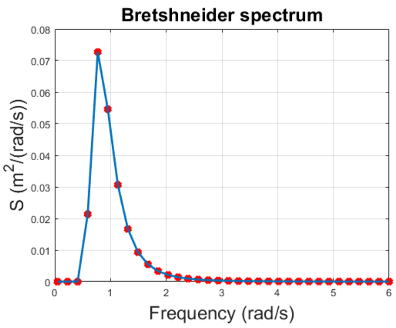


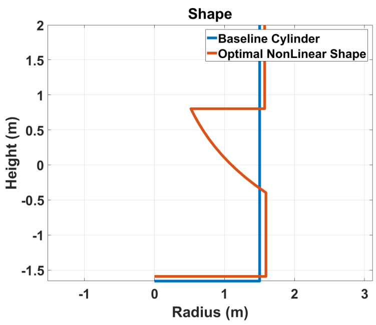
Fig. 3. Bertschneider spectrum used in the time domain simulation. The spectrum is with $H_s = 0.8m$ and $T_p = 10s$.

A. *Optimization emphasize on average power extraction*

The shape and control coefficients were optimized so as to maximize the power quality. Alternatively, to maximize the average power extraction. The optimization problem can be expressed as follows:

$$Maximize : F_{cost} = \left[\int_0^T P(t)dt \right] /T \qquad (12)$$

Simulation results shown as Fig. 4, 5 and 6.



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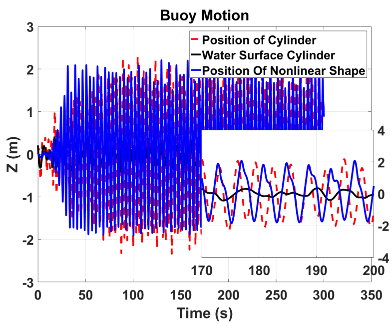
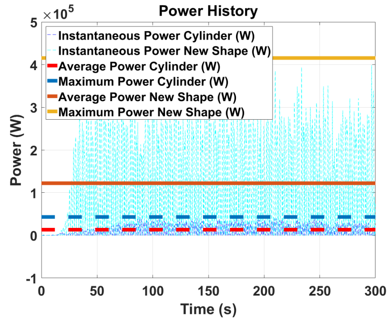


(b)

Fig. 4. Shape comparison between the optimal-average-power solution and the baseline cylindrical WEC. And motion comparison between both cases in time domain simulation.

The new design has a mass of $12054kg = 12.054ton$, similar to the $12000kg$ mass baseline cylindrical buoy. The power quality was good for this design, the average power over a $300s$ simulation was $122100W = 122KW$. But it requires a large control force. And the motion was in the same level with the baseline cylinder buoy.



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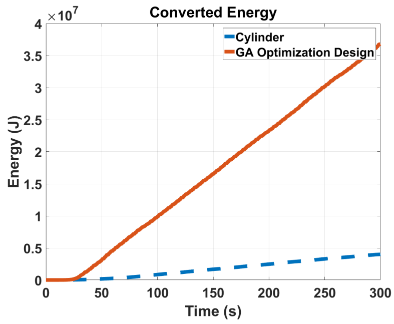

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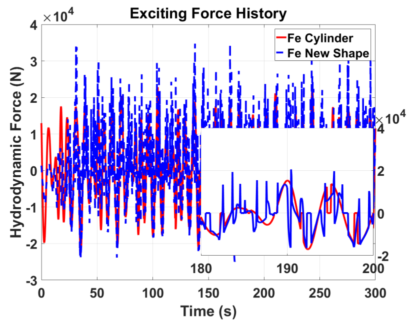
Fig. 5. Time domain simulation results of the optimal-average-power solution and the baseline cylindrical WEC, in terms of instantaneous power, mean power, maximum power, and total converted energy.



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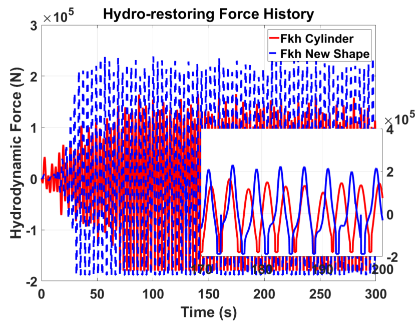


(b)

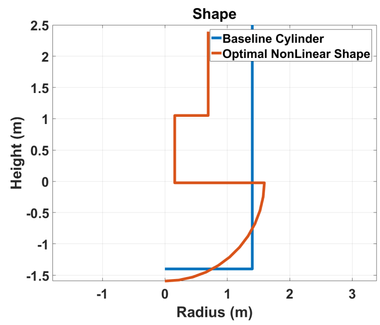
Fig. 6. Different hydrodynamic force history for the optimal-average-power solution and the baseline WEC in the time domain simulation.

B. *Optimization emphasize on maximum energy conversion per unit mass*

The shape and control coefficients were optimized so as to maximize the ratio of the harvest energy over the mass of the buoy. Alternatively, to maximize the energy conversion per unit mass. Material cost was not considered here, as levelized cost function would be more efficient to investigate construction costs in the future. The optimization problem can be expressed as follows:

$$Maximize : F_{cost} == E/m = \left[\int_0^T P(t)dt \right] /m \quad (13)$$

Simulation results shown as Fig.7, 8 and 9.



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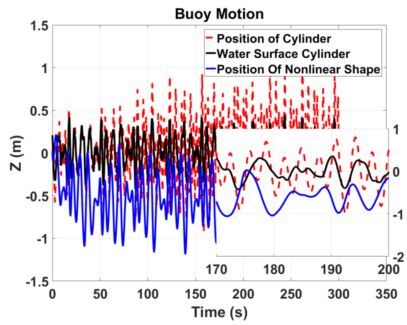


(b)

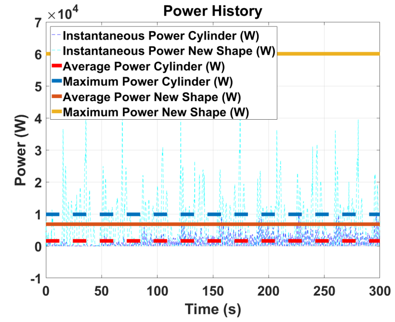
Fig. 7. Shape comparison between the optimal-energy-per-unit-mass solution and the baseline cylindrical WEC. And motion comparison between both cases in time domain simulation.

The new shape has a mass of $9129kg$, similar to the $9200kg$ baseline cylindrical buoy. The energy ratio over a $300s$ simulation was $224.5006J/kg$. However, the power quality was not good as the ratio between the mean and maximum power was small. This implies that this design can not provide consistent high power. As a result of the large control force, the oscillating motion of the buoy was damped with respect to the baseline cylindrical buoy.

V. CONCLUSION

A GA optimization tool is developed to optimize the buoy shape of non-linear axisymmetric WECs and the nonlinear control coefficients simultaneously. The time-domain nonlinear Froude-Krylov force of a complex buoy is computed by decomposing the complex shape into basic shape sections and adopting analytical formulas. The optimization tool is tested in a realistic Bretschneider spectrum wave.

The main findings of this paper are: First, a useful tool is developed to optimize the buoy shapes of WECs under a nonlinear hydrodynamic model. Second, WECs with nonlinear buoy shapes can be more efficient in energy extraction then that with traditional



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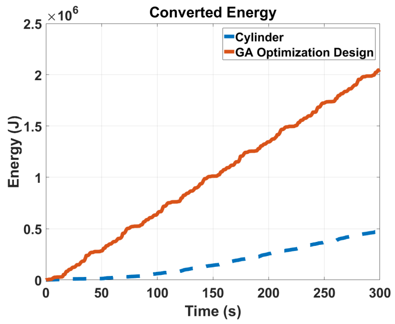

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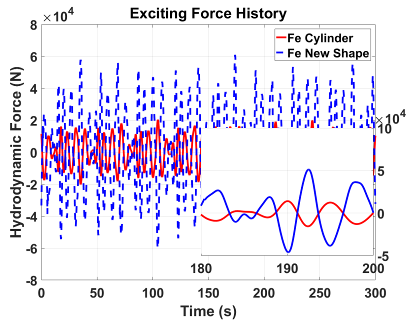
Fig. 8. Time domain simulation results of the optimal-energy-per-unit-mass solution and the baseline cylindrical WEC, in terms of instantaneous power, mean power, maximum power, and total converted energy.



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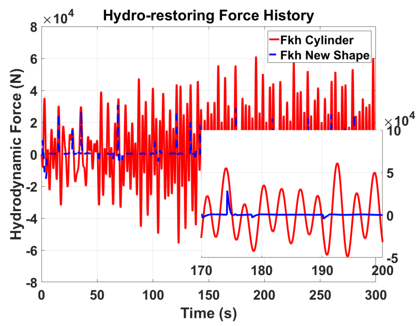

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Fig. 9. Different hydrodynamic force history for the optimal-energy-per-unit-mass solution and the baseline WEC in the time domain simulation.

linear buoy shapes. Third, the nonlinear Froude-Krylov force of a complex WEC buoy can be evaluated analytically. Finally, further optimization of the nonlinear buoy shape can be carried out tuning the cost function of the GA optimization tool, the shape of the buoy can be optimized to meet specific requirements of energy conversion.

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