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Lans-α and Leray turbulence models for coastal simulations: application to Alderney Race

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Abstract—**Alderney race (France) is one of the most promising exploitation site for the tidal stream industry. In this region, where tidal currents reaching to 5m/s, tidal energy estimate requires the use of an accurate turbulence model. The main objectives of this work are to augment a coastal circulation model with the Lagrangian Averaged Navier–Stokes–alpha (LANS-α) and Leray turbulence models in order to study the turbulence structures in Alderney race. In comparison with standard dissipative turbulence models include in MARS (e.g. two equations eddy viscosity turbulence models), the main advantage of LANS-α/Leray is to produce more realistic eddy structures near the grid scale. Firstly, validation of our implementations is performed within an idealised rectangular periodic domain forced by a sinusoidal wind stress and Coriolis parameter. This test shows the ability of LANS-α/Leray to reproduce both horizontal and vertical kinetic energy and patterns obtained at higher-resolution simulations. Finally, the Leray model is applied in the Alderney Race with a fine realistic bathymetry and meteo-oceanic forcings. It appears that Leray model produces higher kinetic and eddy kinetic energy more realistic than MARS with eddy viscosity model.**

Index Terms—**Marine turbulence, Numerical modeling, Large Eddy Simulations, Lagrangian-averaged Navier–Stokes alpha model, Leray model, Alderney-Race**

I. Introduction

TIDAL current represents a significant marine renewable energy source that may be harnessed using tidal turbines. In France, the most important exploitation site is the Alderney Race where tidal currents reaching to 5 m/s. In this mega-tidal environment, an accurate description of the turbulence is important to have the best estimate of the tidal stream production and preventing the mechanical fatigue caused by turbulence on the devices.

In coastal ocean models, the use of DNS (Direct Numerical Simulations) to resolve the full range of turbulent structures is not possible due to a too much computational cost. Turbulence models must be used to parameterize the effects of the unresolved motions on the resolved ones. This means capturing the physical phenomenon of turbulence using a low resolution, by mimicking the effects of the small scales on the larger ones without calculating them explicitly. This is usually done by introducing a statistical average procedure, leading to the Reynolds-Averaged Navier Stokes (RANS) equations or to use a spatial filter, which leads to the Large-Eddy Simulation (LES) equations. Both methods generates an unclosed term which is modeled by dissipative approach or containing a dissipative component. This concept is well-founded, since the role of the small scales, which are being modeled, is to remove the energy generated through non-linear interactions of the large, resolved scales. However, there are some physical scenarios where the dissipation provided by this approach may be excessive and remove turbulent structures generated by nonlinear interactions.

The Lagrangian-Averaged Navier-Stokes alpha (LANS-α) and Leray models are considered as a non dissipative approaches. These models modify nonlinearity to alter the energy transfer, among scales, thereby providing an alternative way to reach closure without enhancing viscosity. The principle feature of the LANS-α model is that the momentum is an advection-diffusion equation for a Lagrangian-averaged velocity $\mathbf{u}$, while the advecting velocity is an Eulerian-averaged velocity $\mathbf{u}_s$. These names arise in the derivation of the LANS-α model, where velocities are averaged along a particle track (Lagrangian) or at a particular location (Eulerian). In the Leray regularization, the flow is convected with a smoothed velocity $\mathbf{u}_s$ leading to reduce the nonlinear effects by an amount governed by the smoothing properties. Note that equations of both models can be rewritten in the LES template with explicit expression of the subgrid stress [1]. These models can then also be interpreted as a part of LES methods.

LANS-α and Leray has been implemented in the POP ocean model and has proven to be a skillful turbulence coastal model [2]–[4]. In the idealised case of a periodic channel flow mimiking the Antarctic Circulpolar Current, the experiments of [2], [3] have showed a remarkable energization of the eddy and mean kinetic energy fields equivalent to doubling the model resolution. This property of LANS-α is especially interesting for eddy permitting coastal modeling.

The purpose of this work is to augment the coastal circulation model MARS [5] to include the LANS-α and Leray turbulence formulations and to investigate

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how they work for Alderney Race with a fine realistic bathymetry and meteo-oceanic forcings. Hereafter MARS-α and MARS-Leray refer, respectively, to our implementation of LANS-α and Leray models in MARS.

II. Turbulence modeling

A. The LANS-α model

The implementation of the LANS-α model in MARS is based on the primitive-equation form [2]

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}_s \cdot \nabla \mathbf{u} + \nabla \mathbf{u}_s^T \cdot \mathbf{u} = \mathbf{f} \times \mathbf{u}_s - \rho^{-1} \nabla \pi + \mathcal{F}, \quad (1a)$$

$$\nabla \cdot \mathbf{u}_s = 0, \quad (1b)$$

where $(u, v, w) = (\mathbf{u}, w)$ and $(u_s, v_s, w_s) = (\mathbf{u}_s, w_s)$ are the rough and smooth velocity, respectively. ∇ is the horizontal gradient, ρ is the density, $\mathbf{f}$ is the Coriolis parameter, g is gravitational acceleration and $\mathcal{F}$ is the diffusion operator acting of the rough velocity. The rough and smooth fluid velocities are related by the Helmholtz operator

$$\mathbf{u} = \mathbf{u}_s - \alpha^2 \Delta \mathbf{u}_s, \quad (2)$$

where α is the smoothing length scale, also called the filter width. The modified pressure π is given by the following relation

$$\pi = p - \rho_0 \frac{1}{2} \parallel \mathbf{u} \parallel - \rho_0 \frac{\alpha^2}{2} \parallel \nabla \mathbf{u} \parallel, \quad (3)$$

where ρ_0 is the reference density and p is the fluid pressure.

B. The Leray regularization

The Leray model is a regularization model, introduce by the great mathematical analyst Jean Leray [6], which is the first to have develop a turbulence model without enhancing viscous dissipation. The model consist on the introduction of a regularized velocity which modifies the non-linear term of the momentum equation to the well-known form [4]

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}_s \cdot \nabla \mathbf{u} = \mathbf{f} \times \mathbf{u} - \rho^{-1} \nabla p + \mathcal{F} \quad (4a)$$

$$\nabla \cdot \mathbf{u} = 0 \quad (4b)$$

There are two differences between the LANS-α and the Leray model: the incompressibility condition in LANS-α is imposed on the smooth velocity $\mathbf{u}_s$, whereas in Leray it is imposed on the rough velocity $\mathbf{u}$. The second difference is that LANS-α momentum equation (1a) has the additional term $\nabla \mathbf{u}_s^T \cdot \mathbf{u}$, leading to ensure the Kelvin's circulation theorem while conserving energy and a form of potential vorticity in the absence of dissipation.

C. Numerical implementation

Implementation of LANS-α and Leray in MARS is ensuring by rewriting (1) in sigma coordinate and by solving the new set of equations using hydrostatic and Boussinesq assumptions. The method of computation is the same for both models, except for the continuity and the addition of the $\nabla \mathbf{u}_s^T \cdot \mathbf{u}$ term. An attractive feature of the Leray model is that it can be easily implemented. The main reason for this is that the continuity equation is in terms of $\mathbf{u}$, instead of $\mathbf{u}_s$ as is the LANS-α. MARS is a semi-implicit code coupling barotropic (vertically integrated 2D) and baroclinic (remaining 3D) velocity fields through an iterative predictor–corrector procedure. This procedure ensures a high numerical stability which is very close to the one of the barotropic model alone. However this approach requires to smooth the velocity during both predictor and corrector step, increasing the computational time. To overcome this difficulty, the smoothing operator (2) is replaced by a convolution filter which have been shown to have the same effects than the Helmholtz operator while being cheaper [3]. The implementation of MARS-α barotropic solver is the part presenting a particularly difficult challenge. The barotropic solver of MARS includes an iterative solution for the surface elevation, and the formal derivation of LANS-α in this algorithm requires smoothing steps within each iteration that is too expensive. To overcome this difficulty, we have designed a reduced formulation that avoids the smoothing within the iterative step. In the simulations presented here, the filter size, representing the stencil size used during the weighted average procedure is fixed as 9×9.

D. Diffusion operator

Although LANS-α and Leray are based of the modification of the non linear term without introduce viscous dissipation, in practice, the use of artificially high viscosity (eddy viscosity) is still required to remove energy near the grid scale in order to avoid numerical instabilities. In addition, LANS-α and Leray are primarily concerned with parameterizing the effects of horizontal mesoscale eddies and does not replace parameterization of oceanic vertical mixing, and may be run in conjunction with this [4]. The diffusion operator $\mathcal{F}$ is decomposed as

$$\mathcal{F} = \nu_H \Delta \mathbf{u} + \nu_V \frac{\partial^2 \mathbf{u}}{\partial z^2}, \quad (5)$$

where ν_V is the vertical mixing and the horizontal viscosity ν_H is expressed as [7]:

$$\nu_H = 0.01 \times \text{fvisc} \times \Delta_H^{0.01}, \quad (6)$$

where Δ_H is the horizontal mesh size and fvisc is an user coefficient ranging between 1 and 17. Note that, the use of hight value of ν_H is necessary, because LANS-α/Leray does not provide a mechanism for energy dissipation near the grid scale.

III. Idealised test case

E. Configuration

In order to validate our implementation, we have designed an idealised test case similar to the one proposed in [2]. The domain is a periodic channel with sidewalls, $L_x = 3200$Km long and $L_y = 1600$Km wide

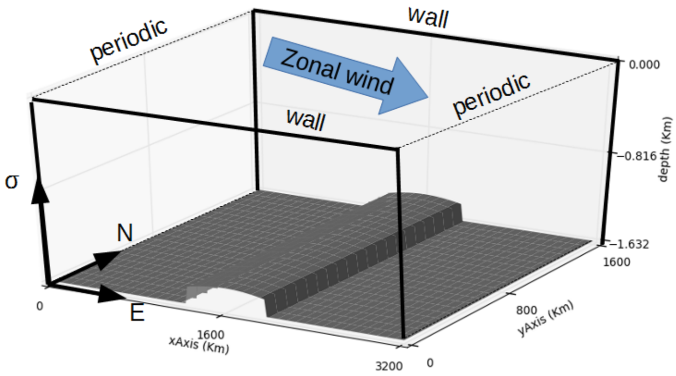

Fig. 1. Schematic of the idealised periodic channel with solid north/south boundaries, an uniform Gaussian-shaped bump, and a surface wind stress forcing.

TABLE I
COMPUTATIONAL TIME (NOTED CT) FOR MARS USING COARSE RESOLUTION (40 KM), FINE RESOLUTION (20 KM) AND MARS-α USING COARSE RESOLUTION. THE RATIO IS CALCULATED BY DIVIDING EACH COMPUTATIONAL TIME BY THE COMPUTATIONAL TIME OF MARS USING COARSE GRID.

Model	MARS (40Km)	MARS (20Km)	MARS-α (40Km)
CT	00H:12mn:27s	01H:46mn:13s	00H:33mn:31s
Ratio	1	8.6472	2.7286

(Fig1). The flow is generated by imposition of a sinusoidal wind stress (τ_{sx}) driven an eastward circulation in the channel:

$$\tau_{sx} = -\tau_0 \cos\left(\pi y / L_x\right), \tag{7}$$

where τ_0 is the wind amplitude. A bump uniform from north to south is placed at centre of the seabed and is typically chosen to be Gaussian-shaped with the form

$$h(x, y) = h_b \exp\left[\left(x - x_c\right)^2 / d^2\right], \tag{8}$$

where $d = 730$Km is the horizontal scale of the bump, $h_b = 0.2$Km is the height that the bump rises above the otherwise flat bottom with depth $h_0 = 1.632$Km and x_0 is the location of the center of the bump, taken here to be the center of the channel ($x_0 = 1600$Km).

For this test, MARS was run for two different horizontal resolutions, referred to as coarse (40 Km) and fine (20Km) grids. At the fine resolution, the turbulent structures are better resolved and the velocity field contain more numerous eddies compared to the coarse resolution. Simulation have been performed during a period of one year with a constant time step of 30mn and using 34 uniform sigma levels. The vertical mixing is fixed as $\nu_V = 10^{-6}\text{Pa}\cdot\text{s}^{-1}$ (no vertical parametrization) and we chose to fix the coefficient fvisc of equation (6) so that ν_H is the same for all simulations at any one resolution so as to allow for unbiased comparison.

F. Results and discussions

When the horizontal resolution of MARS simulation is decreased by a factor of two, mesoscale eddies are better resolved (Fig.2) and the eddy kinetic energy

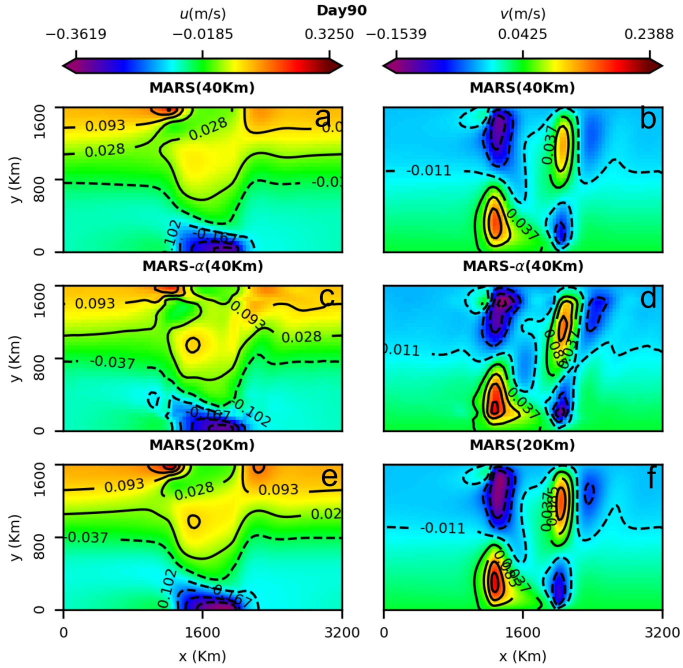

Fig. 2. Instantaneous zonal (a, c, e) and meridional (b, d, f) surface velocity along x and y directions, superposed with their isolines.

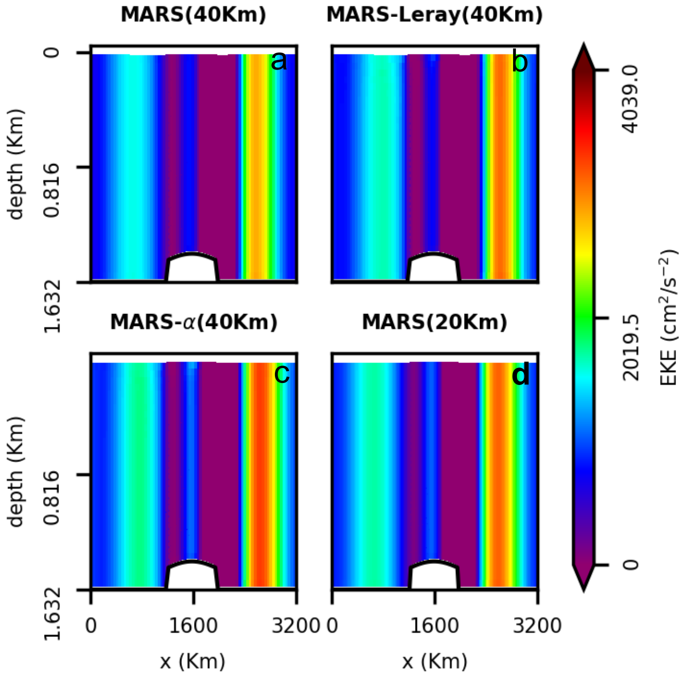

Fig. 3. Vertical section of eddy kinetic energy through the y-axis center at $y = 800$Km using (a) MARS Coarse, (b) MARS-Leray Coarse, (c) MARS-α Coarse, and (d) MARS Fine, in cm^2s^{-2}.

(EKE) is increased (Fig.3). MARS-α simulations with coarse resolution also capture some turbulent structures like simulation at finer rezolution. Fig.3 shows an important property of the LANS-α and Leray models: they produce more eddy activity than MARS and MARS-Leray is less energetic than MARS-α. Statistical resolution effects are also seen in the EKE using LANS-α and Leray : EKE averaged over the domain increases with resolution, and lower resolution MARS-α and MARS-Leray simulations also capture this effect. As presented in TABLE I, the additional computational cost due to MARS-α is only a factor 2.73, versus a factor of 8.65 for doubling the horizontal resolution of MARS.

IV. Alderney Race

After the idealised test case, we proceeded to a Alderney Race simulations. We focussed our discussion on the creation and motion of two gyres which have been observed and described in [8].

G. Nesting strategy

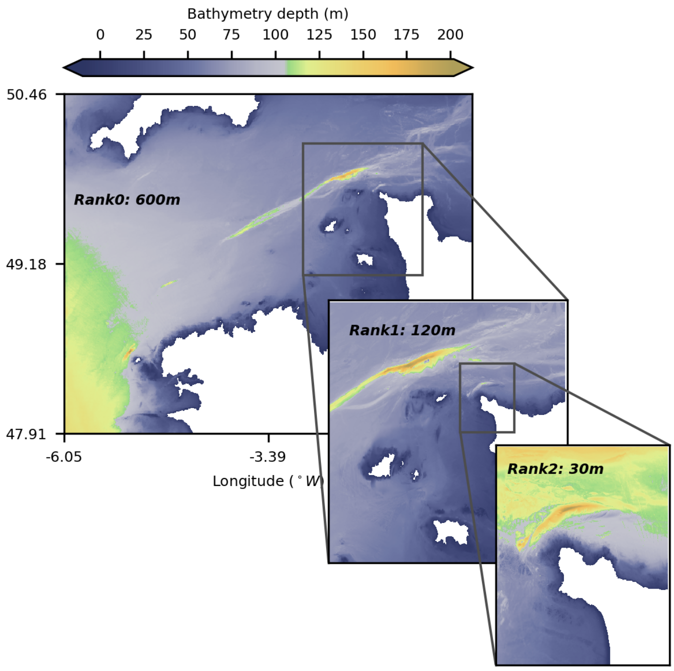

Fig. 4. Grid nested approach and bathymetry variations associated at each grid.

Simulations have been performed involving embedded grids (called ranks) as presented in Fig.4. Bathymetry resolution is the order of a meter [9]. Starting from a broad region (i.e. rank 0) which extends in longitude from -6 °W to 50.4 °N in latitude (with 600 m of horizontal resolution) down to a detailed domain (i.e. rank 2) covering a few kilo meters and which extends in longitude from -2 ° W to 49.84° N in latitude. The rank 0 is forced by water levels providing by the SHOM (Hydrological and Oceano-graphic Service of the French Navy) [10] while rank1 and rank2 receive their boundary conditions from rank0 and rank1 simulations, respectively.

H. Simulation parameters

Simulations have been performed during a period of one day starting from the 17th October 2012 at 12h:00mn:00s where we have a tidal coefficient of 109. All ranks are three-dimensional with 20 uniform sigma levels. We use a constant time step set to Δt =5s for the ranks 0, 1 and Δt =2s for the rank2. The vertical mixing term ν_V of equation (6) is calculated using the well-known turbulent closure $k-\epsilon$ modified according to [11] and based of the generic length scale approach [12].

I. Results and discussions

Our implementation of LANS-α for realistic application is in progress, and is not yet complete. The very rough seabed and the strong tidal currents make the Alderney Race the theater of hightly energetic turbulent structure and is then more subject to instability than the idealised channel. Here we present, only the results from MARS-Leray simulations because numerical instabilities are produced with MARS-α. As presented in F, MARS-Leray is less energetic than MARS-α but remains more energetic than MARS. As presented in Fig5, MARS-Leray ran stably in the Alderney Race and has higher kinetic energies than MARS, when the comparison is made at the same resolution of 30m. Fig5 also shown the creation and motion of two gyres circulation surrounded in red and green. These gyres are clearly more energetic and better captured with MARS-Leray (Fig5 (a) and (b)). The gyres move in the North-East direction and merge to generated a single structure surrounded in red(Fig5 (c)). Fig5 (d) shows that the MARS simulation loses to capture the motion of this gyre which is not the case of Leray. These turbulent structures have been observed and described in [8]. They drive the motions of the radioactive tracers ejected from the AREVA site in La Hague. So, it is very useful for the community to be able to reproduce them accurately. In the future, comparisons with observations described in [8] will be performed.

V. Conclusion

We have presented the implementation of LANS-α and Leray turbulence models within the coastal model MARS. Our experiments with an idealised test case have shown the augmentation of LANS-α and Leray in MARS leads to achieve eddy kinetic energy that is near to or greater than that of a doubled MARS rezolution. Zonal and meridional patterns also become more realistic as eddy activity increases, showing that LANS-α properly captures the turbulent structures using a coarse rezolution. The Leray model improves turbulence statistics in the same manner as LANS-α, but to a lesser degree. The addition of either parameterization to MARS adds about a factor 2.73 to the computational time, so it is a small expense compared to a doubling of resolution, which increases computational time by a factor of 8.75

Application of LANS-α to Alderney Race with the complex rough seabed and strong tidal currents is not yet complete due to numerical instabilities. Simulations with the Leray model show interesting results: higher kinetic energy and better capture of circulation gyres.

Simulations of Leray model in Alderney Race has been obtained using the turbulent closure $k-\epsilon$ to calculate the vertical mixing term. Future work will focus on how to optimally combine LANS-α and Leray with vertical mixing parameterizations, with a validation against ADCP datas and stabilize LANS-α for application in Alderney Race.

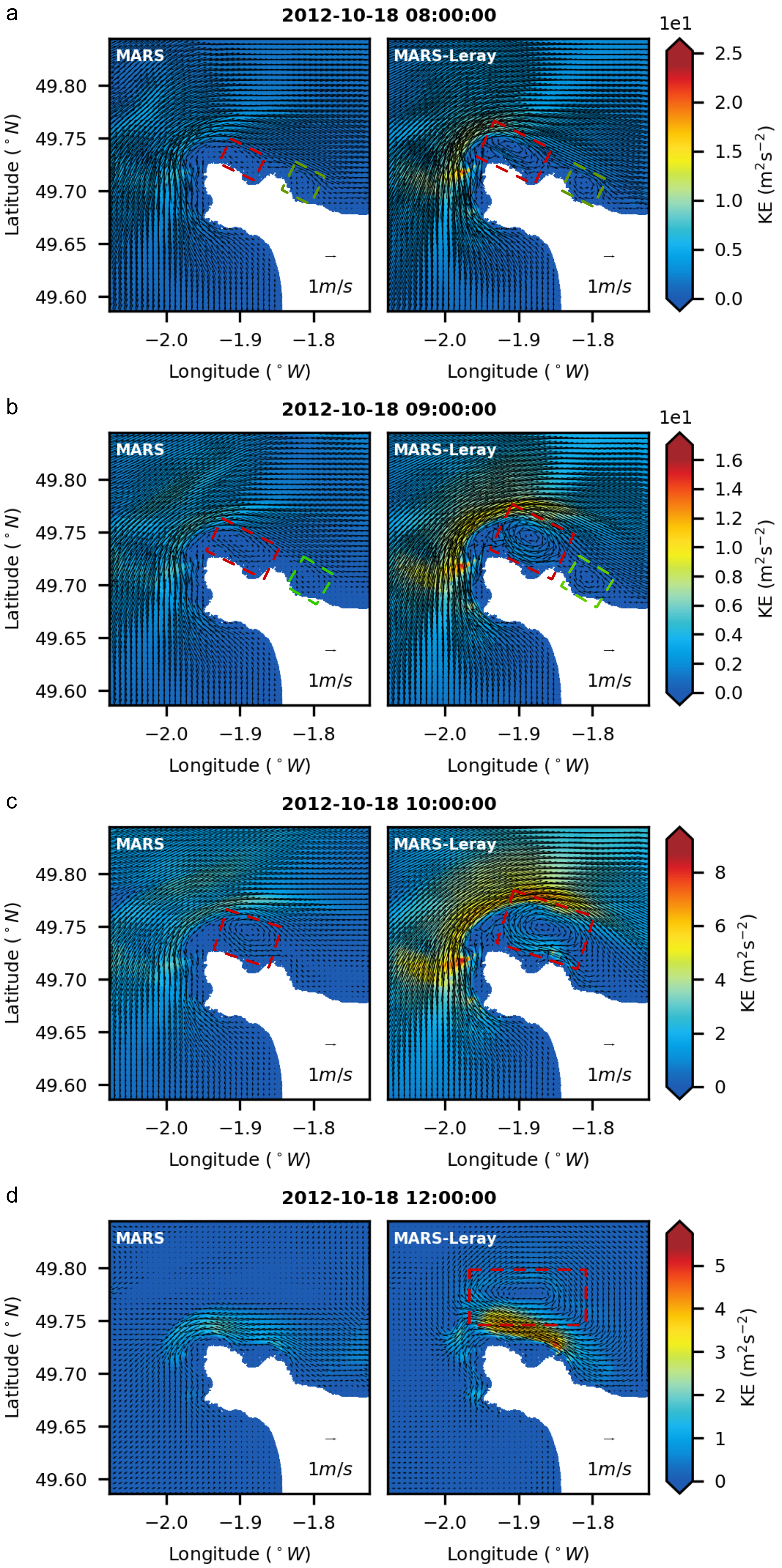

Fig. 5. Time evolution of the surface kinetic energy (KE) superposed with its corresponding velocity vector for MARS (left row) and MARS-Leray (right row).

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