Proceedings of the 13th European Wave and Tidal Energy Conference 1-6 Sept 2019, Naples, Italy

Weakly nonlinear theory of an array of surging wave energy converters with curved geometry

Simone Michele, Emiliano Renzi and Paolo Sammarco

Abstract—**We analyse the effect of gate surface curvature on the nonlinear dynamics of an array of surge-type wave energy converters (WECs) in a channel. Using an asymptotic expansion up to the third order, we show the occurrence of new detuning and damping coefficients in the Ginzburg-Landau evolution equation, which are not present in the case of flat surging WECs. The occurrence of nonlinear synchronous resonance by monochromatic incident waves with small amplitude and frequency equal to the trapped-mode eigenfrequency is shown. Unlike the case of flat WECs or linear theories, synchronous excitation is now possible because of interactions between the wave field and the curved WEC's surface at higher orders. Finally, the effects of the power take-off (PTO) device on the generated power and the global response of the array are studied. We obtain that nonlinear synchronous resonance can be substantial for design purposes.**

Keywords—**Resonance, wave-structure interaction, nonlinear theory.**

I. Introduction

In this paper we examine the synchronous nonlinear resonant excitation of an array of curved surge-type wave energy converters (WECs) in a semi-infinite long channel. Surge-type WECs are among the most efficient and promising devices to capture energy from ocean waves. Such bodies oscillate under the action of incident waves and are capable of producing energy with potentially large efficiency. For an extensive review we refer to [1]-[2]. The literature on the hydrodynamic modelling of surge-type devices is vast and deeply developed by several authors [3]-[11]. Works on experimental analysis [12] and numerical investigations [13] are available as well.

A large part of the theoretical models developed so far on the dynamics of surging WECs neglect nonlinear interactions between water waves and oscillating bodies. This can be unjustified when trapped array modes are resonantly excited by normally incident waves with small amplitude.

ID number: 1371; Conference track: WHM.

The work of S. Michele and E. Renzi is supported by a Royal Society - CNR International Fellowship.

S. Michele and E. Renzi are with the Department of Mathematical Sciences, Loughborough University, Leics LE11 3TU, UK.

P. Sammarco is with the department of Civil Engineering and Computer Science, University of Rome 'Tor Vergata', Via del Politecnico 1, 00133 Roma, Italy.

Indeed, [14] recently showed that subharmonic resonance and mode competition of trapped modes significantly increase energy extraction efficiency of an array of gate-type devices. Furthermore, recent studies on curved flap-type gates suggest that using curved structure could further improve the economics of WECs by maximizing wave power extraction in non-resonant configurations [15]. Motivated by these new developments, in this work we analyze the effect of gate surface curvature on the synchronous nonlinear resonant dynamics of an array of surging WECs.

We show that a small horizontal deviation of the gate surface produces significant changes in the behaviour of the system. Using an asymptotic expansion up to the third order, the nonlinear governing equations is decomposed in a sequence of linearized boundary-value problems of order n and harmonic m [3]. Gate shape effects resonate the first harmonic at the second order, so that three timing with a slow time scale and a super-slow time scale is necessary to avoid secularity [16].

Here we consider the synchronous excitation of a single mode by monochromatic waves with small amplitude and frequency corresponding to the eigenfrequency of the trapped mode. A similar phenomenon can be found in the context of edge waves [17]. Products between the gate shape function and the second-order terms that force the first harmonic at the third order are obtained. We derive the corresponding complex nonlinear evolution equation of the Ginzburg-Landau type [18]-[19], which describes the time evolution of the resonated trapped mode amplitude. Such an equation is more complicated than that already studied by [14], because it includes new terms which depend on the shape array function. We then characterise the occurrence of stable and unstable branches for the equilibrium fixed points and show that large damping removes instability.

We remark that this nonlinear excitation is not possible for flat WECs, because in that case the evolution equation would be damped and unforced. We finally show that synchronous resonance yields constructive interactions that can be substantial for design purposes.

II. Nonlinear Governing Equations

With reference to Fig.1, consider a semi-infinite channel of constant depth h' and width b'. Define a Cartesian reference system (x', y', z') with the horizontal x' and y'-axes lying on the undisturbed free surface level and the z' axis pointing vertically upward. Primes above

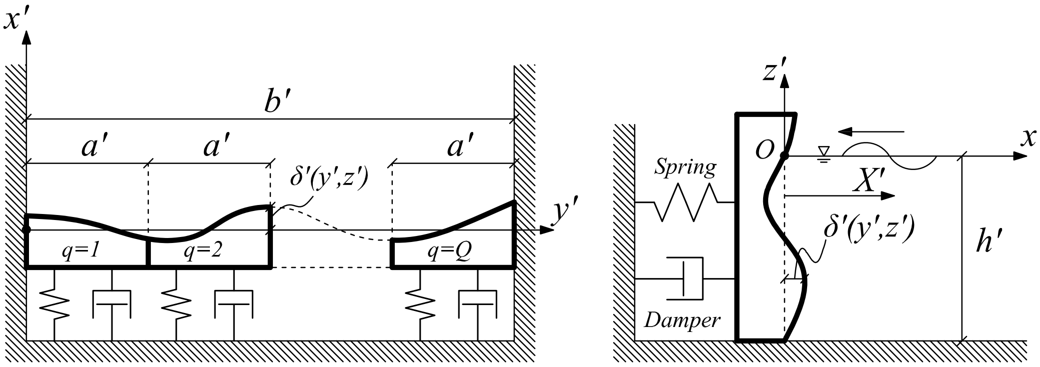

Fig. 1. Plan geometry and side view of the system. Each body is connected with a vertical back wall through a spring-damper (PTO) system in parallel.

the variables indicate physical quantities. At $x' = 0$ rests an array of surge-type WECs, each with mass M' and width a', allowed to oscillate horizontally (surge) along the channel without bottom friction, under the action of incident monochromatic waves.

The choice of a surging WEC device simplifies the algebra but does not change the mathematical structure of problems involving, for example, devices installed in a breakwater. In this last case, the breakwater need to be properly designed to trigger nonlinear resonances.

Each WEC is connected to the channel back wall by a spring-PTO/damper system operating in parallel. The spring has elastic constant C', while the PTO has constant coefficient ν'_{pto} and exerts a force proportional to the gate horizontal velocity. Let us assume incoming waves from $x' \to +\infty$, normally incident to the gates. Let G_q, *q=1,…,Q,* denotes the qth gate and X'_q be the horizontal displacement of G_q in the x' direction. Then we can define $X'(y',t') = \{X'_1(t'), \dots, X'_Q(t')\}$ as the piece-wise displacement function of the entire array. The fluid is assumed to be inviscid and incompressible and the flow irrotational. Hence, the velocity potential $\Phi(x',y',z',t')$ satisfies the Laplace equation in the fluid domain *Ω*. The position of the wetted gate surface is described by the following equation

$$x' - X' - \delta' = 0, \tag{1}$$

where δ' denotes the deviation of the array surface from the vertical plane $x' = 0$. Let A'_T be the scale of the free-surface trapped oscillations, λ' the wavelength, ω' the eigenfrequency of the natural mode and g' the acceleration due to gravity. Then introduce the following non-dimensional quantities:

$$\begin{gathered}(x,y,z) = \frac{(x',y',z')}{\lambda'}, G = \frac{g'}{\omega'^2\lambda'}, \\ \Phi = \frac{\Phi'}{A'_T\lambda'\omega'}, \zeta = \frac{\zeta'}{A'_T}, t = \omega' t', \\ (a,b,h) = \frac{(a',b',h')}{\lambda'}, X = \frac{X'}{A'_T}, \delta = \frac{\delta'}{\delta'_g},\end{gathered} \tag{2}$$

where ζ' is the free surface elevation, δ'_g the length scale for δ' and G the non-dimensional eigenfrequency of order $O(1)$. Let the following length ratios be much smaller than unity:

$$\varepsilon = A'_T/\lambda' \sim \delta'_g/\lambda' \ll 1. \tag{3}$$

Since the partial derivatives of δ' with respect to the coordinates y', z' are of order $O(\epsilon)$, the latter assumptions imply that the shape of the array must be smooth and regular.

Using the dimensionless variables (2)-(3), we derive the following governing equation, nonlinear boundary conditions and equation of motion in non-dimensional form. The Laplace and Bernoulli equations in the fluid domain are, respectively,

$$\nabla^2\Phi = 0, \tag{4}$$

$$-\frac{p'}{\rho'\omega'^2\lambda'} = Gz + \varepsilon\frac{\partial\Phi}{\partial t} + \varepsilon^2\frac{1}{2}|\nabla\Phi|^2, \tag{5}$$

where ρ' is the fluid density. The dynamic and mixed boundary conditions on the free surface are given by

$$-G\zeta = \frac{\partial\Phi}{\partial t} + \varepsilon\frac{1}{2}|\nabla\Phi|^2, \qquad z = \varepsilon\zeta, \tag{6}$$

$$\begin{gathered}\frac{\partial^2\Phi}{\partial t^2} + G\frac{\partial\Phi}{\partial z} + \varepsilon\frac{\partial|\nabla\Phi|^2}{\partial t} + \varepsilon^2\frac{1}{2}\nabla\Phi\cdot\nabla|\nabla\Phi|^2 \\ = 0, \qquad z = \varepsilon\zeta,\end{gathered} \tag{7}$$

while the no-flux conditions at the bottom and channel walls require

$$\frac{\partial\Phi}{\partial z} = 0, \qquad z = -h, \tag{8}$$

$$\frac{\partial\Phi}{\partial y} = 0, \qquad y = 0, \qquad y = b. \tag{9}$$

The kinematic condition on the non-dimensional time-dependent array surface $x = \varepsilon(X + \delta)$ can be written as

$$\frac{\partial\Phi}{\partial x}=\frac{\partial X}{\partial t}+\varepsilon\left(\frac{\partial\Phi}{\partial y}\frac{\partial\delta}{\partial y}+\frac{\partial\Phi}{\partial z}\frac{\partial\delta}{\partial z}\right), \tag{10}$$

while the equation of motion of the qth curved gate coupled with a PTO device is given by

$$\begin{aligned}\varepsilon M\frac{\partial^2 X_q}{\partial t^2}+\varepsilon SGX_q+\varepsilon^3\nu_{pto}\frac{\partial X_q}{\partial t}\\=\int_{(q-1)a}^{qa}dy\left\{\int_{-1}^{\varepsilon\zeta}dz\left(\varepsilon\frac{\partial\Phi}{\partial t}\right.\right.\\\left.\left.+\varepsilon^2\frac{1}{2}|\nabla\Phi|^2\right)+\int_{-1}^{\varepsilon\zeta}Gzdz\right\},\end{aligned} \tag{11}$$

where M=$M'/(\rho'\lambda'^3)$ is the non-dimensional mass of the gate, S=$S'/(g'\rho'\lambda'^2)$ the non-dimensional stiffness of the spring, ν_{pto}=$\nu'_{pto}/(A'^2_T\omega'\rho'\lambda')$ the non-dimensional PTO coefficient while all the terms on the right-hand side represent the contribution due to the dynamic pressure. In (11) we have assumed the effect due to the PTO damping force on the gate motion to be small if compared to the other terms. Large values of ν'_{pto} comparable with leading-order terms would render the equation of motion at *O(1)* damped and unforced, so that a trapped-mode solution would not be possible anymore. Assuming the following scales: $A'_T\sim O(1)$ m, $\omega'\sim O(1)$ rad s^{-1}, $\lambda'\sim O(10)\div O(10^2)$ m, the values of ν'_{pto} that satisfy the scale above are of order $O(10^4)\div O(10^5)$ Kg s^{-1}.

Now we demonstrate that these orders of magnitude are physically congruent with power take-off systems for practical engineering applications. If the array moves in-phase, solution of the linearized two-dimensional radiation velocity potential yields the radiation damping acting on each gate

$$\bar{\nu}'=\frac{\omega'\rho'a'(\sinh k'h')^2}{k'^3}\left(\frac{2k'h'+\sinh 2k'h'}{4k'}\right)^{-1}, \tag{12}$$

where k' is the real solution of the dispersion relation

$$\omega'^2=g'k'\tanh k'h'. \tag{13}$$

Maximum power extraction efficiency requires resonance and $\bar{\nu}'=\nu'_{pto}$ [3], hence expression (12) gives a first estimate for the optimal damping that maximizes generated power for a resonated surging WEC. If the gate dimensions are of order $O(10)$ m, expression (12) yields $\bar{\nu}'\sim O(10^5)$ Kg s^{-1}, i.e. a value comparable with the PTO-coefficient scaling assumed in this work. This means that nonlinear effects due to hydrodynamic contributions can be important because they are comparable with the PTO damping term. Thus, neglecting nonlinear hydrodynamic terms might cause one to overlook constructive resonance phenomena like the synchronous resonance mechanisms analysed here.

III. Perturbation Harmonic Expansion and Three Timing

Let us assume the following expansions for the unknowns up to the third order $O(\varepsilon^2)$:

$$\begin{aligned}\{\Phi,\zeta,X\}=\\=\sum_{m=0}^{1}\{\phi_{1m},\eta_{1m},\chi_{1m}\}e^{-imt}\\+\sum_{m=0}^{2}\varepsilon\{\phi_{2m},\eta_{2m},\chi_{2m}\}e^{-imt}\\+\sum_{m=0}^{1}\varepsilon^2\{\phi_{3m},\eta_{3m},\chi_{3m}\}e^{-imt}+*,\end{aligned} \tag{14}$$

where the symbol $*$ denotes the complex conjugate of the terms inside the series. We point out that the unknowns above depend on the slow-time scales $t_1=\varepsilon t$ and $t_2=\varepsilon^2 t$, respectively. Unlike the case analysed in [14], the three-timing assumption is necessary here because of the presence in the governing equations of terms representing the shape of the array. These terms give a resonant forcing for the first-harmonic solution at the second and third order, so that solvability conditions must be applied in order to avoid secularities. Moreover, the three-time scales add terms in the evolution equation for the modal amplitude at the third order $O(\varepsilon^2)$ and the corresponding stability analysis for the equilibrium states increases in complexity.

Substitution of (14) into the governing equation, boundary conditions and equation of motion (4)-(11) allows us to split the nonlinear problem in a sequence of linear boundary value problems of order n and harmonic m. Corresponding forcing terms and methods of solutions are algebraically quite long and are not reported here for the sake of brevity. Here we refer to [18] for mathematical insights.

Here the synchronous excitation of a single trapped mode by small monochromatic incident waves at the second order $O(\varepsilon)$ is analysed. We remark that such nonlinear dynamics is peculiar to curved gates and nonlinear resonance mechanisms. Indeed, trapped modes cannot be resonated in linear theories by normally incident waves because of orthogonality between the modal matrix and forcing terms, while flat systems do not allow nonlinear synchronous excitation when the incident waves are small if compared to the trapped wave field. This is due to the absence of first-harmonic terms at the third order that include the forcing incident wave potential. In that case, the evolution equation would be damped and unforced and the corresponding solution would be given by the trivial stable state.

IV. Nonlinear Synchronous Resonance

The first harmonic problem at the leading order is homogeneous and unforced, hence the corresponding solution is given by the trapped mode only [7]. At the second order, the inhomogeneous problem is forced by products between the first order solution and the gate shape function. Since ω and ϕ_{11} solve the $O(1)$ boundary value problem, while the second order-first harmonic problem for ϕ_{21} is forced, a solvability condition must be now applied to ϕ_{11} and ϕ_{21} to avoid secularity. Let us return back in physical variables except for the slow time scales t_1 and t_2. After some lengthy algebra, Green's theorem over the entire fluid domain yields the following complex evolution equation

$$\frac{\partial \chi'}{\partial t_1} - \frac{i c_\delta}{\omega' \delta'} \chi' = 0, \tag{15}$$

where χ' is the amplitude of the trapped mode, while the real coefficient c_δ represents a modulation of the modal amplitude growth depending on gate curvature and the first-order trapped mode solution. The evolution equation (15) is linear, thus the corresponding solution is given by

$$\chi'(t_1, t_2) = \vartheta'(t_2) e^{-i c_\delta t_1}. \tag{16}$$

Note that flat gates yield $c_\delta = 0$, so in this case χ' depends on the super-slow time scale t_2 only (see also [14]). At the third order, the inhomogeneous problem is forced by second and first order solutions, respectively. For the same reasons of the first harmonic problem at the second order analysed before, we invoke the solvability condition by applying Green's theorem to ϕ'_{11} and ϕ'_{31} over the fluid domain Ω. After some lengthy algebra we obtain the following forced evolution equation of the Ginzburg-Landau form [19] for the modal amplitude depending on the slow time scale t_2:

$$\begin{aligned} -\varepsilon^2 i \frac{\partial \vartheta'}{\partial t_2} = \vartheta'(c_A + i c_B) + \vartheta'^2 \vartheta'^*(c_N + i c_R) \\ + A' e^{\frac{-i c_\delta t_1}{\varepsilon}}(c_S + i c_U) \\ + i \vartheta' \nu'_{pto} c_L, \end{aligned} \tag{17}$$

where ϑ'^* is the complex conjugate of ϑ'. The latter equation now has additional terms when compared to the evolution equation in [14] for flat WECs. These are the new terms c_A and c_B, the complex forcing coefficient (c_S+ic_U) and the real coefficient c_δ.

The coefficients c_A and c_B represent, respectively, detuning and damping caused by the shape of the array. Flat gates ($\delta' = 0$) give $c_A = c_B = 0$. We point out that c_A and c_B are invariant for profiles that are symmetrical about $x' = 0$. The forcing coefficients c_S and c_U represent the energy influx by the incident waves. Is it possible to demonstrate that both flat and symmetrical configurations about the vertical plane $y' = b'/2$ yield $c_S = c_U = 0$ and that cannot be resonated synchronously.

Concerning the other coefficients, c_N represents nonlinearity, c_R is the radiation damping due to the second-harmonic radiation at the second order while c_L represents the effects due to the linear PTO.

Instead of perfect resonance, let us consider a detuning $\Delta\omega$ between the trapped mode and the incident wave frequency such that the ratio $\Delta\omega'/\omega' = \omega_2 \varepsilon^2$ [14]. Then, the evolution equation (17) becomes

$$\begin{aligned} -i \frac{\partial \bar{\vartheta}'}{\partial t'} = \bar{\vartheta}'(\Delta\omega' + c_A + i c_B) \\ + \overline{\vartheta'^2} \bar{\vartheta}'^*(c_N + i c_R) \\ + A'^{(c_S + i c_U)} + i \bar{\vartheta}'^{\nu'}{}_{pto} {}^{c_L}, \end{aligned} \tag{18}$$

where

$$\vartheta' = \bar{\vartheta}' e^{-i\left(\omega_2 t_2 + \frac{c_\delta t_1}{\omega' \varepsilon}\right)}. \tag{19}$$

Since the corresponding energy equation is given by

$$\begin{aligned} \frac{\partial |\bar{\vartheta}'|^2}{\partial t'} = -2|\bar{\vartheta}'|^2 (c_B + \nu'_{pto} c_L) - 2|\bar{\vartheta}'|^2 c_R \\ - 2A' \operatorname{Im}\{(c_S - i c_U)\bar{\vartheta}'\} \end{aligned} \tag{20}$$

and both coefficients c_R and $(c_B + \nu'_{pto} c_L)$ are positive, their effect is to damp the modal amplitude. Usage of action-angle variables $\bar{\vartheta}' = R' e^{i\psi'}$ yields a nonlinear dynamical system in the unknowns R' and ψ'

$$\begin{aligned} \frac{\partial R'}{\partial t'} = -R'(c_B + \nu'_{pto} c_L) - R'^3 c_R + \\ A'(c_S \sin\psi' - c_U \cos\psi'), \end{aligned} \tag{21}$$

$$\begin{aligned} \frac{\partial \psi'}{\partial t'} = \Delta\omega' + R'^2 c_N + \frac{A'}{R'}(c_U \sin\psi' + \\ c_S \cos\psi'). \end{aligned} \tag{22}$$

The latter system admits nontrivial stable/unstable states while trivial fixed points corresponding to $R' = 0$ are absent. In particular, nontrivial fixed points are given by the roots of the following implicit equation in R':

$$\begin{aligned} -R'(\nu'_{pto} c_L + c_B + R'^2 c_R) + \\ \sqrt{A'^2(c_S^2 + c_U^2) - R'^2(c_N R'^2 + \Delta\omega' + c_A)^2} = \\ 0, \end{aligned} \tag{23}$$

for which corresponding solutions can be obtained through numerical methods. Equation (23) admits either a single stable fixed point or the coexistence of three roots, i.e. two stable points and one unstable saddle. This depends on the values of both detuning $\Delta\omega'$ and PTO coefficient ν'_{pto}. Differently from what was obtained by [14], the stable branch never coincides with the origin, hence the amplitude at the equilibrium is always positive.

V. POWER EXTRACTION EFFICIENCY

Let us consider the simplest case of $Q = 2$ gates. This system has two degrees of freedom, thus solution of the homogeneous first-harmonic problem at the leading

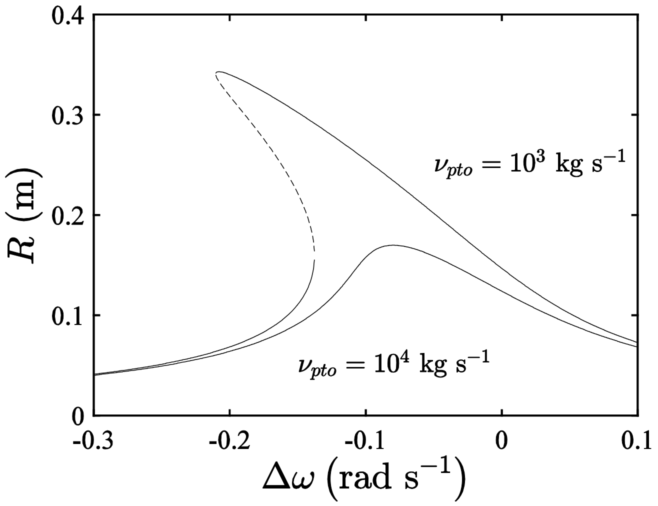

Fig. 2. Stable and unstable equilibrium branches for the first configuration δ'_1. The solid lines correspond to stable fixed points, while the dotted line is related to an unstable saddle.

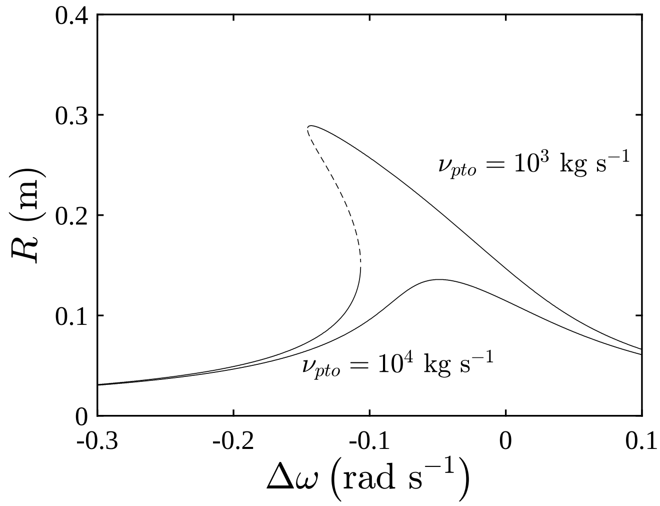

Fig. 3. Stable and unstable equilibrium branches for the second configuration δ'_2.

order yields a single out-of-phase trapped mode with normalized eigenvector $r_q = \{-1, 1\}$.

Let us consider constant water depth $h' = 5$ m and gate width $a' = 5$ m. The amplitude of the small incident waves must be at the second order, thus we assume $A' = 0.1$ m. Now, let us compare two different gate configurations that can be of practical engineering interest, respectively:

$$\delta'_1 = \frac{b'}{10}\cos\frac{\pi y'}{b'}, \tag{24}$$

$$\delta'_2 = \frac{b'}{10}\cos\frac{\pi y'}{b'}\frac{\cosh 0.24(h'+z')}{\cosh 0.24h'}. \tag{25}$$

The number 0.24 (in m^{-1}) inside δ'_2 denotes the wavenumber corresponding to the eigenfrequency $\omega' = 1.4$ rad s^{-1}. This value has been chosen to check whether matching of vertical eigenfunctions and gate profile gives constructive resonant effects in terms of generated power.

Let us focus the attention on a fixed eigenfrequency and analyse the effects of the PTO damping coefficient on the dynamic behaviour. For example, take $\omega' = 1.2$ rad s^{-1} and assume two values of ν'_{pto}, respectively 10^3 and 10^4 kg s^{-1}. The corresponding equilibrium branches defined by (23) for configurations δ'_1 and δ'_2 are plotted in Fig. 2 and Fig. 3. The continuous lines correspond to stable fixed points, while the dashed line is related to unstable saddles. Note that for large values of ν'_{pto} the unstable fixed point disappears, thus we have one stable fixed point for the entire range of detuning $\Delta\omega'$ and absence of non-trivial instability.

The corresponding capture factor, which assesses the system's efficiency, reads [5]

$$C^F = \frac{2\nu'_{pto}(\omega'+\Delta\omega')^2\sum_{q=1}^{Q} r_q^2 R'^2}{E'C'_g b'}, \tag{26}$$

where the term at the numerator of (26) is the generated power by the array, the denominator represents the energy flux by the incident waves per array width b, while

$$E'C'_g = \frac{\rho' g' A'^2(\omega'+\Delta\omega')}{4k'}\left(1+\frac{2k'h'}{\sinh 2k'h'}\right). \tag{27}$$

In the latter equation, both group celerity C'_g and wavenumber k' are related to the frequency $(\omega'+\Delta\omega')$.

Fig. 4 and Fig. 5 show the maximum of C^F for the configurations δ'_1 and δ'_2, respectively. The maximum value is 0.7 for δ'_1. This means that a device designed to resonate synchronously trapped modes can achieve significant efficiency. Fig.5 also shows that C^F has a smooth trend without maxima or minima in correspondence of $\omega' = 1.4$ rad s^{-1}. Therefore, matching the gate profile to the vertical eigenfunction does not give significant contributions. Finally note that configuration δ'_1 is more efficient than δ'_2. This is because the forcing coefficients c_S and c_U for the first configuration are always greater than the coefficients for configuration 2, hence forcing contributions over δ_1 are greater as well.

VI. Conclusion

We analysed the hydrodynamic interactions between an array of curved surge-type WECs and weakly nonlinear waves in a semi-infinite channel. Perturbation expansion and three-timing with two slow-time scales allowed us to find the complex evolution equations of the Ginzburg-Landau for synchronous excitation. New damping, detuning and forcing coefficients that are dependent on the array shape function and its derivatives appear in the equation. This dynamic is possible only for nonlinear theories, because forcing contributions now depend on the products between the gate shape function and second order velocity potentials at the third order.

Then the effects of curved shapes on the synchronous resonance mechanism are investigated by comparing two WEC configurations of practical engineering interest. We

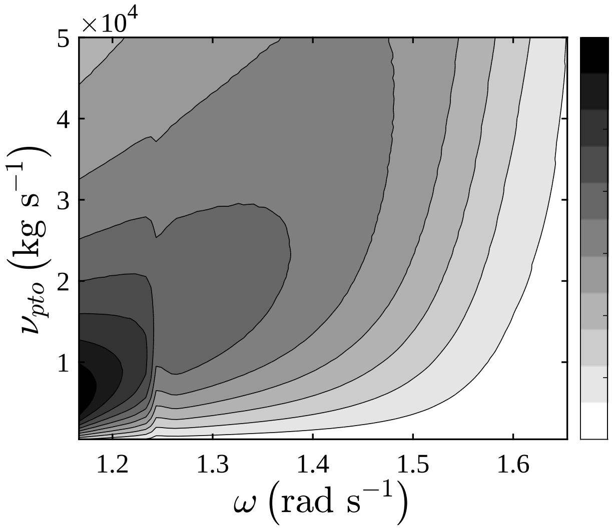

Fig. 4. Behaviour of the capture factor C^F for the first configuration δ'_1. versus the eigenfrequency ω for different values of ν'_{pto}. The maximum value is 0.7, i.e. a value significant for design purposes.

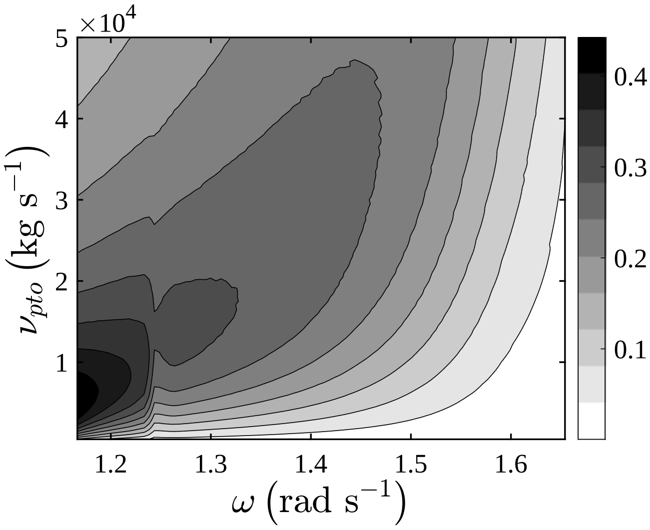

Fig. 5. Behaviour of the capture factor C^F for the second configuration δ'_2. versus the eigenfrequency ω' for different values of ν'_{pto}. The overall efficiency is lower than the case for δ'_1.

found that effects of synchronous interactions on the generated power can be substantial for optimization purposes. This highlights the importance of including nonlinear resonances in the cost-benefit analysis when choosing the gate shape.

We also demonstrated that the PTO damping term at the third order is physically coherent with systems for power absorption and that can be comparable with higher-order hydrodynamic terms.

We assumed the PTO coefficient be the same for each gate and constant in time. Variable values modify the equation of motion at the third order, and the mathematics increases in complexity. This is an interesting aspect that will be investigated in the close future.

Moreover, in this work we analysed the case of normally incident waves. Oblique waves of small amplitude excite normal modes at higher orders and remove the symmetry in the equation of motion at the second order. This can be important if the WECs are in real seas conditions where the direction of the incoming waves is not necessarily orthogonal to the array surface.

Finally, damping effects such fluid shear stresses and vortex shedding between adjacent gates are inevitable in real conditions. These detrimental effects reduce the gate oscillations and the corresponding generated power and should be considered to better evaluate the efficiency of the WEC array.

REFERENCES

[1] A. Babarit, Ocean wave energy conversion. Elsevier, 2018

[2] F. Dias, E. Renzi, S. Gallagher, D. Sarkar, Y. Wei, T. Abadie, C. Cummins and A. Rafiee, "Analytical and computational modelling for wave energy systems: the example of oscillating wave surge converters," *Acta Mech. Sin.*, vol. 33, pp. 647–662, 2017.

[3] C. C. Mei, M. Stiassnie and D. K.-P. Yue, Theory and application of ocean surface waves. Singapore: World Scientific, 2005.

[4] C. Linton and P. McIver, Mathematical techniques for wave/structure interactions. Chapman & Hall/CRC, 2001.

[5] E. Renzi and F. Dias, "Resonant behaviour of an oscillating wave energy converter in a channel," *J. Fluid Mech.*, vol. 701, pp. 482–510, 2012.

[6] E. Renzi and F. Dias, "Hydrodynamics of the oscillating wave surge converter in the open ocean," *Eur. J. Mech. B/Fluids*, vol. 411, pp. 1–10, 2013.

[7] P. Sammarco, S. Michele and M. d'Errico, "Flap gate farm: from Venice lagoon defense to wave energy production. Part 1: natural modes," *Appl. Ocean Res.*, vol. 43, pp. 203–213, 2013.

[8] S. Michele, P. Sammarco, M. d'Errico, E. Renzi, A. Abdolali, G. Bellotti and F. Dias, "Flap gate farm: from Venice lagoon defense to wave energy production. Part 1: synchronous response to incident waves in open sea," *Appl. Ocean Res.*, vol. 52, pp. 43–61, 2015.

[9] S. Michele, P. Sammarco and M. d'Errico, "The optimal design of a flat gate array in front of a vertical wall: resonance of the natural modes and enhancement of the exciting torque," *Ocean Eng.*, vol. 118, pp. 152–164, 2016.

[10] S. Michele, P. Sammarco and M. d'Errico, "Theory of the synchronous motion of an array of floating flap gates oscillating wave surge converter," *Proc. R. Soc. London A.*, vol. 472, 20160174, 2016.

[11] I. Noad and R. Porter, "Optimisation of arrays of flap-type oscillating wave surge converters," *Appl. Ocean Res.*, vol. 50, pp. 237–253, 2015.

[12] L. Wilkinson, T. Wittaker, P.R. Thies, S. Day and D. Ingram, "The power-capture of a nearshore, modular, flap-type wave energy converter in regular waves," *Ocean Eng.*, vol. 137, pp. 394–403, 2017.

[13] E. Renzi and F. Dias, "Motion resonant modes of large articulated damped oscillators in waves," *J. Fluid Struct.*, vol. 49, pp. 705–715, 2014.

[14] S. Michele, P. Sammarco and M. d'Errico, "Weakly nonlinear theory for oscillating wave surge converters in a channel," *J. Fluid Mech.*, vol. 834, pp. 55–91, 2018.

[15] S. Michele, E. Renzi and P. Sammarco, "A second-order theory for wave energy converters with curved geometry," in *33rd International Workshop on Water Waves and Floating Bodies*, Guidel-Plages, France, 2018.

[16] A. H. Nayfeh and D. T. Mook, Nonlinear oscillations. New York: Wiley Classic Library, 1995.
[17] P. Blondeaux and G. Vittori, "The nonlinear excitation of synchronous edge waves by a monochromatic wave normally incident on a plane beach," *J. Fluid Mech.*, vol. 301, pp. 251–268, 1995.
[18] S. Michele, E. Renzi and P. Sammarco, "Weakly nonlinear theory for a gate-type curved array in waves," *J. Fluid Mech.*, vol. 869, pp. 238–263, 2019.
[19] P. G. Drazin, Introduction to hydrodynamic instability. United Kingdom: Cambridge University Press, 2002.