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Adaptive Collective Control using Consensus Estimation in Arrays of Wave Energy Converters

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Abstract—**The adaptive Collective Control using Consensus Estimation is proposed in this paper. The performance of the Wave Energy Converters (WEC) array is optimized by the Collective Control (CC) in terms of the energy extraction. The physical limitation is also considered in deriving the control. The Continuous-Discrete Kalman Consensus Filter (CDKCF) is implemented to estimate the required information for the controller. The proposed estimator utilize the communication in the hydrodynamic coupled WEC array to improve the estimation and deploy the estimation in a distributed fashion. The unforced WEC array is first simulated. The results show CDKCF has a good performance in estimating the system behavior and the wave field. Additionally, the CDKCF is compared with the regular Kalman Filter (KF). The results indicate the CDKCF improves the estimation both in the estimation error and the disagreement. Second, the simulation is conducted on the forced WEC array. The performance shows the adaptive CC can optimize the energy production of the WEC array with satisfying the constraints. The extracted energy is around 95% of the energy produced by the CC with perfect knowledge of the wave field. We can conclude the proposed control has a good performance in terms of energy production and the Consensus estimation guarantees the controller is adaptive to the constantly-changing sea condition.**

Index Terms—**Wave Energy Conversion, Wave Energy Converters Array, Collective Control, Consensus Estimation, Adaptive Control**

I. Introduction

Among different sustainable energy resources, the wave energy is considered to be more reliable and has higher energy density. In 1970s', the wave energy conversion is studied by some pioneers in references [1]–[4] regarding the hydrodynamics and control. The force oscillation hydrodynamic model is widely applied in the community which provides the convenience for the control design. The complex conjugate control is developed in reference [5], in which the wave energy conversion is analyzed in the frequency domain and the theoretical maximum energy production is predicted. Many controllers are developed for single point absorber. Reference [6] developed the singular arc control which is shown to be comparable with the complex conjugate control. By considering the physical constraints, reference [7] introduced the Model Predictive Control which can exactly include the constraints into control design. Alternatively, reference [8], [9] proposed the Pseudo-spectral approach which implements the constraints in the numerical optimization.

The research on the wave energy conversion is further extended to the WEC array. The hydrodynamics of the WEC array is studied in references [10]–[12]. Nowadays, the Boundary Element Methods (BEM) approach are widely applied in solving the hydrodynamics. Numerous software are developed on that purpose such as WAMIT [13], ANSYS® AQWA and Nemoh [14]. In this paper, WAMIT is applied to solve the hydrodynamics of the WEC array. The layout of the WEC array has a significant impact on the performance of the array. As shown in reference [15], [16], the triangular shape is desired. Hence, in this paper, the three floaters in the array will be placed in a triangle. The constructive effect of interaction in the WEC array motivates the researchers to study the control of the WEC array. Reference [17] investigated the coordinated control and found the coordinated control is significantly better than the independent control regarding energy extraction. Additionally, the global control is studied in reference [18] which is found to offset the destructive interaction in the array. References [19], [20] proposed the Model Predictive Control of the WEC array by considering the state and force constraints. In this paper, the Collective Control developed in reference [21] is applied and optimized to maximize the energy absorption and satisfy the physical limitation.

The proposed control requires the current and future information of the wave field and states. Hence, an estimator is required to estimate and predict that information. Although there are many wave estimators are developed in references [22]–[25] for a single WEC. In the WEC array, the hydrodynamics of each buoy is strongly coupled, but each buoy will only collect local measurement and estimate the wave field and system behavior of the entire array. Consequently, a consensus estimator is desired for this objective which can fuse the data in a distributed fashion and is believed to have a more accurate estimation. The Kalman-Consensus Filter (KCF) is proposed in references [26], [27]. Reference [28] further extended the KCF to CDKCF for engineering application. The CDKCF is applied in this paper to estimate the wave field and WEC array's behavior. The estimated quantities will be fed into the control prediction horizon to calculate the current control which is accordingly adaptive to the changing

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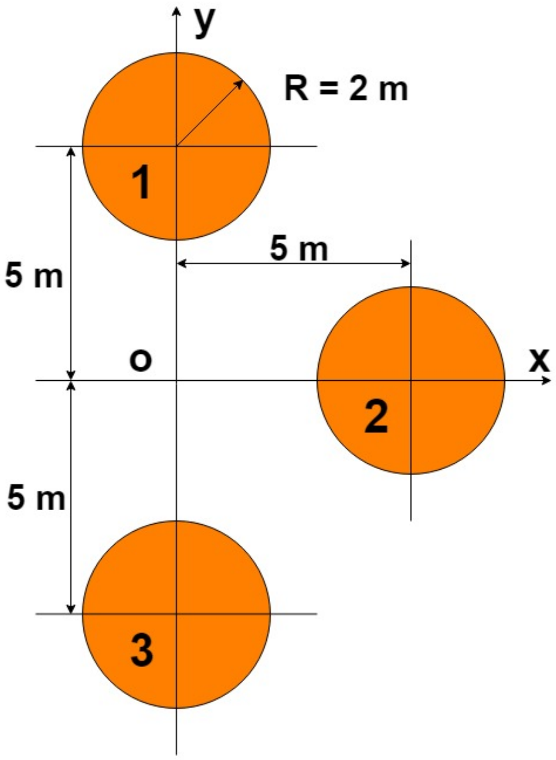

Fig. 1: The layout of the WEC array

sea condition.

This paper presents a novel adaptive CC using consensus estimator. The contribution of this work is: first, the CC is improved to be adaptive by applying the Consensus estimation. Second, the Consensus estimation provides an accurate estimation which guarantees the optimality of the controller. The derivation of the hydrodynamics, control, and consensus estimation are introduced in Section. II. The numerical simulation is presented in Section. III. Section. IV draws the conclusions.

II. Mathematical Derivations

In this section, we will present the mathematical modeling of the hydrodynamics of the WEC array. The CDKCF and CC will also be derived. As shown in Fig. 1, there are three buoys in the triangular WEC array with the coordinates $(0, 5)$, $(5, 0)$ and $(0, -5)$. Each buoy in the WEC array has a radius of 2m and a rigid body mass of 1.7174×10^4kg. Additionally, the wave direction in the array is along the positive x direction ($\beta = 0^o$) and only the heave motion is considered.

A. The hydrodynamics

The hydrodynamics of the WEC array can be described based on the Cummins' Equation [29] as:

$$\begin{aligned} \dot{\vec{v}} &= \mathbf{M}^{-1}(\vec{F}_e + \vec{u} - \mathbf{K}\vec{z} - \mathbf{C}_r\vec{x}_r) \\ \dot{\vec{x}}_r &= \mathbf{A}_r\vec{x}_r + \mathbf{B}_r\vec{v} \end{aligned} \tag{1}$$

where $\vec{z}$ and $\vec{v}$ represent the displacement and velocity of the buoys in the array. $\vec{x}_r$ is the radiation states vector. $\mathbf{M}$ is the total mass matrix which can be computed as $\mathbf{M} = \mathbf{m}_r + \mathbf{m}_\infty$, where:

$$\mathbf{m}_r = diag([m_{r,11},\, m_{r,22},\, m_{r,33}]) \tag{2}$$

$$\mathbf{m}_\infty = \begin{bmatrix} m_{\infty,11} & m_{\infty,12} & m_{\infty,13} \\ m_{\infty,21} & m_{\infty,22} & m_{\infty,23} \\ m_{\infty,31} & m_{\infty,32} & m_{\infty,33} \end{bmatrix} \tag{3}$$

The $\mathbf{K}$ is the hydrostatic restoring force coefficient matrix which can be expressed as $\mathbf{K} = diag([K_{11},\, K_{22},\, K_{33}])$. Moreover, the $\vec{F}_e$ is the excitation force vector where the ith element can be computed as:

$$F_{e,i} = \sum_{n}^{N_\omega} \Re(F_{e,c,0}(\omega_n)\eta_c(\omega_n)e^{j(k_n r_i - \omega_n t + \phi_n)}) \tag{4}$$

where $i = 1, 2, 3$. The N_ω is the total number of frequencies of the ocean wave. $F_{e,c,0}$ is the excitation force coefficients at the origin of the WEC array which can be extracted from WAMIT [13]. η_c is the frequency dependent wave elevation. ω_n is the frequency and ϕ_n is the random time domain phase shift. k_n is the wave number which can be expressed as $k_n = \frac{\omega_n^2}{g}$. The $\mathbf{A}_r$, $\mathbf{B}_r$ and $\mathbf{C}_r$ in the equation of motion (Eq. (1)) are the radiation matrices which can be calculated based on the radiation damping and added mass obtained from WAMIT. The $\vec{u}$ is the control force. The hydrodynamics can be written in a state space format as:

$$\begin{aligned} [\dot{x}_1,\, \dot{x}_2,\, \dot{x}_3]^T &= [x_4,\, x_5,\, x_6]^T \\ [\dot{x}_4,\, \dot{x}_5,\, \dot{x}_6]^T &= \mathbf{M}^{-1}(\vec{F}_e + \vec{u} - \mathbf{K}\vec{z} - \mathbf{C}_r\vec{x}_r) \\ \dot{\vec{x}}_r &= \mathbf{A}_r\vec{x}_r + \mathbf{B}_r[x_4,\, x_5,\, x_6]^T \end{aligned} \tag{5}$$

where $x_{1,2,3} = z_{1,2,3}$ is the displacement of buoy $1, 2, 3$, $x_{4,5,6} = v_{1,2,3}$ is the velocity of buoy $1, 2, 3$. The state vector is defined as $\vec{x} = [x_1,\, x_2,\, x_3,\, x_4,\, x_5,\, x_6,\, \vec{x}_r]^T$.

B. The Collective Control

To optimize the energy extraction of the WEC array, the CC is applied in this paper. The detailed derivation can be found in reference [21]. The CC applies the Proportional-derivative (PD) feedback control law:

$$\vec{u} = -\mathbf{K}_p[x_1,\, x_2,\, x_3]^T - \mathbf{K}_d[x_4,\, x_5,\, x_6]^T \tag{6}$$

where:

$$K_p = diag([K_{p,11},\, K_{p,22},\, K_{p,33}]) \tag{7}$$

$$K_d = \begin{bmatrix} K_{d,11} & K_{d,12} & K_{d,13} \\ K_{d,21} & K_{d,22} & K_{d,23} \\ K_{d,31} & K_{d,32} & K_{d,33} \end{bmatrix} \tag{8}$$

The $\mathbf{K}_p$ and $\mathbf{K}_d$ are optimized such that the WEC array can produce the maximum energy within the constraints. Hence the objective function can be described as:

$$\begin{aligned} Min: \quad & J = -E + \frac{1}{2} r_g \sum_{i=1}^{3} (max(0, g_i))^2 \\ & K_{d,ii} \geq K_{d,ij},\ i \neq j,\ i, j = 1,\, 2,\, 3 \end{aligned} \tag{9}$$

where $E = \sum_i \int (-u_i v_i) dt$ is the energy extraction of the array. $g_i = \mid z_i \mid - z_{max}$ is the inequality constraints on the displacement of ith buoy. Additionally, the control force cannot exceed the maximum control capacity $|\ u_i\ | < u_{max}$, so the control force will be saturated by u_{max}. The cost function will be minimized by applying Matlab® fmincon function. During the optimization, the state vector $\vec{x}$ will be propagated using Eq. (5). The weight of the penalty function r_g will be tuned to guarantee the performance of the WEC array satisfy the constraints without losing optimality.

C. The Continuous-Discrete Kalman Consensus Filter

To evaluate the CC control, the information of the displacement, velocity, radiation states and excitation force field are required. Hence the CDKCF is developed in this section to estimate those quantities. The details of the derivation and initialization of CDKCF can be found in reference [28]. The excitation force field in the dynamics can be approximated as:

$$\begin{aligned} F_{e,i} &\approx \sum_{n=1}^{\tilde{N}_\omega} a_n \cos(k_n r_i - \omega_n t) + b_n \sin(k_n r_i - \omega_n t) \\ &\approx \boldsymbol{\phi}(r_i, t)[\vec{a},\, \vec{b}]^T \end{aligned} \tag{10}$$

where $\boldsymbol{\phi}(r_i, t) = [\cos(\vec{\theta}_i),\, \sin(\vec{\theta}_i)]$ and $\vec{\theta}_i = \vec{k}r_i - \vec{\omega}t$. The dynamics of the estimator can be expressed as:

$$\begin{aligned} [\dot{\hat{x}}_1,\, \dot{\hat{x}}_2,\, \dot{\hat{x}}_3]^T &= [\hat{x}_4,\, \hat{x}_5,\, \hat{x}_6]^T \\ [\dot{\hat{x}}_4,\, \dot{\hat{x}}_5,\, \dot{\hat{x}}_6]^T &= \mathbf{M}^{-1}(\boldsymbol{\phi}(\vec{r}, t)[\hat{\vec{a}},\, \hat{\vec{b}}]^T + \vec{u} - \\ &\quad \mathbf{K}[\hat{x}_1,\, \hat{x}_2,\, \hat{x}_3]^T - \mathbf{C}_r \hat{\vec{x}}_r) \\ \dot{\hat{\vec{x}}}_r &= \mathbf{A}_r \hat{\vec{x}}_r + \mathbf{B}_r [\hat{x}_4,\, \hat{x}_5,\, \hat{x}_6]^T \\ \dot{\hat{\vec{a}}} &= \vec{0} \\ \dot{\hat{\vec{b}}} &= \vec{0} \end{aligned} \tag{11}$$

where

$$\boldsymbol{\phi}(\vec{r}, t) = \begin{bmatrix} \boldsymbol{\phi}(r_1, t) \\ \boldsymbol{\phi}(r_2, t) \\ \boldsymbol{\phi}(r_3, t) \end{bmatrix} \tag{12}$$

The estimated state vector is $\hat{\vec{x}} = [\hat{x}_1,\, \hat{x}_2,\, \hat{x}_3,\, \hat{x}_4,\, \hat{x}_5,\, \hat{x}_6,\, \hat{\vec{x}}_r,\, \hat{\vec{a}},\, \hat{\vec{b}}]^T$. From the dynamics, we can tell the coefficients of the excitation force approximation $\vec{a}$ and $\vec{b}$ are not updating in the propagation. However they are not fixed during the estimation, the coefficients will be adjusted based on the measurements. The estimated states are the same for all the buoys in the WEC array which means each buoy will estimate the information of the entire array:

$$\hat{\vec{x}}_i = \hat{\vec{x}} \quad i = 1,\, 2,\, 3 \tag{13}$$

Accordingly, the estimated state is a consensus among different agents in the WEC array. Further, the dynamics of the estimator can be rewritten in the form of:

$$\dot{\hat{\vec{x}}}_i = \mathbf{F}\hat{\vec{x}}_i + \boldsymbol{\Gamma}\vec{u} \tag{14}$$

where

$$\mathbf{F} = \begin{bmatrix} \mathbf{0}^{N_a \times N_a} & \mathbf{I}^{N_a \times N_a} & \mathbf{0}^{N_a \times N_r} & \mathbf{0}^{N_a \times 2\tilde{N}_\omega} \\ -\mathbf{M}^{-1}\mathbf{K} & \mathbf{0}^{N_a \times N_a} & -\mathbf{M}^{-1}\mathbf{C}_r & \mathbf{M}^{-1}\boldsymbol{\phi} \\ \mathbf{0}^{N_r \times N_a} & \mathbf{B}_r & \mathbf{A}_r & \mathbf{0}^{N_r \times 2\tilde{N}_\omega} \\ \mathbf{0}^{\tilde{N}_\omega \times N_a} & \mathbf{0}^{\tilde{N}_\omega \times N_a} & \mathbf{0}^{\tilde{N}_\omega \times N_r} & \mathbf{0}^{\tilde{N}_\omega \times 2\tilde{N}_\omega} \\ \mathbf{0}^{\tilde{N}_\omega \times N_a} & \mathbf{0}^{\tilde{N}_\omega \times N_a} & \mathbf{0}^{\tilde{N}_\omega \times N_r} & \mathbf{0}^{\tilde{N}_\omega \times 2\tilde{N}_\omega} \end{bmatrix} \tag{15}$$

and

$$\boldsymbol{\Gamma} = \begin{bmatrix} \mathbf{0}^{N_a \times N_a} \\ \mathbf{M}^{-1} \\ \mathbf{0}^{N_r \times N_a} \\ \mathbf{0}^{\tilde{N}_\omega \times N_a} \\ \mathbf{0}^{\tilde{N}_\omega \times N_a} \end{bmatrix} \tag{16}$$

where in those two matrices. N_a represents the number of buoys in the array, N_r is the number of radiation states. $\tilde{N}_\omega$ is the number of frequencies in the estimation. Since the true number of frequencies of the wave field (N_ω) is not measurable. Hence we assume we have no knowledge of the wave frequency distribution, except we can predict a peak frequency based on the long term observation of the wave field. So, for the estimation, we can select a certain number of frequencies ($\tilde{N}_\omega$) distributed around the peak frequency. Typically, to guarantee the efficiency of the estimation, the $\tilde{N}_\omega$ will be much smaller than N_ω, but the wave characteristic still will be captured by the estimation with less number of frequencies. The measurements of each buoy are the displacement and velocity. Hence, the measurement model can be expressed as:

$$\tilde{\vec{y}}_i = \mathbf{H}_i \vec{x}_i + \vec{v} \tag{17}$$

where $\tilde{\vec{y}}_i = [\tilde{z}_i,\, \tilde{v}_i]^T$ contains the displacement and velocity measurements of the ith buoy. The measurement model $\mathbf{H}_i$ is described as:

$$\begin{aligned} &\mathbf{H}_i = \mathbf{0}^{2 \times (2N_a + N_r + 2N_\omega)} \\ &\mathbf{H}_i(1, i) = 1 \quad \mathbf{H}_i(2, N_a + i) = 1 \end{aligned} \tag{18}$$

Furthermore, $\vec{v} \sim (0, \mathbf{R}_i)$ represents the measurement noise and $\mathbf{R}_i$ is the covariance matrix for the measurement noise of ith buoy. The process of the consensus estimation is summarized in Algorithm. 1.

In this algorithm, $\mathbf{P}_i$ is the error covariance matrix of ith buoy. $\mathbf{K}_{k,i}$ is the Kalman Gain of ith buoy at kth stage. γ is the consensus gain. $\boldsymbol{\Upsilon}_i$ is the process noise gain matrix and $\mathbf{Q}$ is the process noise covariance matrix.

D. The Collective Control with Consensus Estimation

Since, the Consensus estimator has collected the required information of the control, the control force can be evaluated as:

$$\vec{u} = -\mathbf{K}_p[\bar{\hat{x}}_1,\, \bar{\hat{x}}_2,\, \bar{\hat{x}}_3]^T - \mathbf{K}_d[\bar{\hat{x}}_4,\, \bar{\hat{x}}_5,\, \bar{\hat{x}}_6]^T \tag{19}$$

where the estimated states are extracted from the average estimation $\bar{\hat{\vec{x}}}$ which can be computed as: $\bar{\hat{\vec{x}}} = \frac{1}{N_a}\sum_i^{N_a} \hat{\vec{x}}_i$. The control coefficients are optimized within the prediction horizon based on the objective

Algorithm 1 Continuous-Discrete Kalman-Consensus Filter: a Continuous-Discrete observer with a consensus term

1: **Initialization:** $\mathbf{P}_i = \mathbf{P}_{i,0}, \vec{x}_i = \vec{x}_{i,0}$
2: **while** new data exists **do**
3: Compute the Kalman Gain $\mathbf{K}_{k,i} = \mathbf{P}^-_{k,i}\mathbf{H}^T_{k,i}(\mathbf{R}_{k,i} + \mathbf{H}_{k,i}\mathbf{P}^-_{k,i}\mathbf{H}^T_{k,i})^{-1}$
4: Update the current estimation based on consensus law $\hat{\vec{x}}^+_{k,i} = \hat{\vec{x}}^-_{k,i} + \mathbf{K}_{k,i}(\tilde{\vec{y}}_{k,i} - \mathbf{H}_{k,i}\hat{\vec{x}}^-_{k,i}) + \gamma\mathbf{P}^-_{k,i}\sum_{N_i}(\hat{\vec{x}}^-_{k,j} - \hat{\vec{x}}^-_{k,i})$ $\mathbf{P}^+_{k,i} = (\mathbf{I} - \mathbf{K}_{k,i}\mathbf{H}_{k,i})\mathbf{P}^-_{k,i}$
5: Propagate the estimation $\dot{\hat{\vec{x}}}_i = \mathbf{F}\hat{\vec{x}}_i + \mathbf{\Gamma}\vec{u}$ $\dot{\mathbf{P}}_i(t) = \mathbf{F}(t)\mathbf{P}_i(t) + \mathbf{P}_i(t)\mathbf{F}^T(t) + \mathbf{\Upsilon}_i(t)\mathbf{Q}(t)\mathbf{\Upsilon}^T_i(t)$
6: **end while**

function defined in Eq. (9). The initial guess of the coefficients can be described as:

$$\mathbf{K}_{p,0} = diag([0.9(\mathbf{M}(1,1)\omega_p^2 - \mathbf{K}(1,1)) \times \mathbf{1}^{(N_a\times 1)}])$$
$$\mathbf{K}_{d,0} = \begin{bmatrix} 4\times 10^5 & 4\times 10^4 & 4\times 10^4 \\ 4\times 10^4 & 4\times 10^5 & 4\times 10^4 \\ 4\times 10^4 & 4\times 10^4 & 4\times 10^5 \end{bmatrix} \tag{20}$$

where $\omega_p^2 \sim max(\sqrt{\bar{\hat{a}}^2 + \bar{\hat{b}}^2})$ which is computed corresponding to the maximum estimated excitation force magnitude. The excitation force field predicted over the horizon is constructed as:

$$\hat{\vec{F}}_{e,i} = \boldsymbol{\phi}(r_i, (t \sim t + T_h))[\bar{\hat{a}}, \bar{\hat{b}}]^T \tag{21}$$

The logic of the adaptive CC with CDKCF is described in Fig. 2. As shown in the figure, the measurement will be fed into the consensus estimator to adjust the estimation. The estimation will be inputted to the collective control to optimize the $\mathbf{K}_p$ and $\mathbf{K}_d$ in the prediction horizon which has a length of T_h. To improve the efficiency of the controller, the control coefficients will not be optimized at each sampling time, a jump time $T_{jp} < T_h$ is defined such that the control coefficients will be optimized at $t = NT_{jp}$ where $N = 1, 2, 3...$, at the other time, the existing coefficients will be applied to compute the control force.

III. Simulation Results

In this section, we will present the simulation results of the CDKCF and CC of the WEC array. The wave applied in the simulation has a Bretschneider spectrum with a significant height of 1m and a peak period of 11s. The disagreement shown in this section can be computed as:

$$\Phi(\hat{\vec{x}}) = (\sum_i^{N_a}\sum_j^{M}\delta_{ij}^2)^{\frac{1}{2}} \tag{22}$$

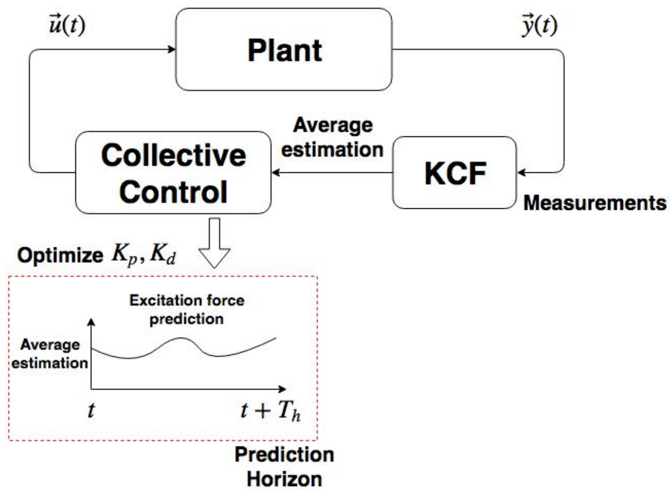

Fig. 2: The logic of the Collective Control with Consensus estimation

where M is the number of elements in $\vec{\delta}_i$. $\vec{\delta}_i = \hat{\vec{x}}_i - \bar{\hat{\vec{x}}}$. Additionally, the average estimation error of the excitation force is computed as:

$$e_{F_e}(\%) = \frac{1}{N_a}\sum_i^{N_a} \frac{|\ \bar{\hat{F}}_{e,i} - F_{e,i}\ |}{max\ |\ \vec{F}_{e,i}\ |} \times 100 \tag{23}$$

The simulation is first conducted on the CDKCF estimation of the coupled, unforced WEC array. Then the performance of the CC is numerically examined with the array information estimated by the CDKCF.

A. The coupled unforced WEC array

In this case, the WEC array is allowed to have a free motion (without control). The motion of all the buoys in the array and the wave field will be estimated by CDKCF. The true wave field has 300 frequencies, and we apply 40 frequencies in the estimation. The consensus gain $\gamma = 3\times 10^{-9}$. The details of the standard deviation of the process noise and the measurement noise are:

$$\sigma_{q,\vec{z}} = 0.02 \quad \sigma_{q,\vec{v}} = 0.04 \quad \sigma_{q,\vec{x}_r} = 5 \quad \sigma_{q,\vec{a},\vec{b}} = 200$$
$$\sigma_{r,\vec{z}} = 0.02 \quad \sigma_{r,\vec{v}} = 0.04 \tag{24}$$

The average estimation of the displacement of buoy $1, 2, 3$ are presented in Fig. 3, 4 and 5. From the figures, we can tell the CDKCF has a good estimation of the motion of the buoys and the estimation converges around 10s. The excitation force field estimation is shown in Fig. 6, 7 and 8. The proposed estimator is able to catch the wave characteristic in the WEC array with less number of frequencies in the estimation. The percentage estimation error of the excitation force with CDKCF is compared with KF (shown in Fig. 9, 10 and 11). In the figures, the estimation error of CDKCF is computed based on the average estimation ($\bar{\hat{\vec{a}}}$ and $\bar{\hat{\vec{b}}}$), however the estimation error of KF is computed based on individual estimation ($\hat{\vec{a}}_i$ and $\hat{\vec{b}}_i$). The regular KF implemented in each buoy estimates $\hat{\vec{x}} = [\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4, \hat{x}_5, \hat{x}_6, \hat{\vec{x}}_r, \hat{\vec{a}}, \hat{\vec{b}}]$. The KF only

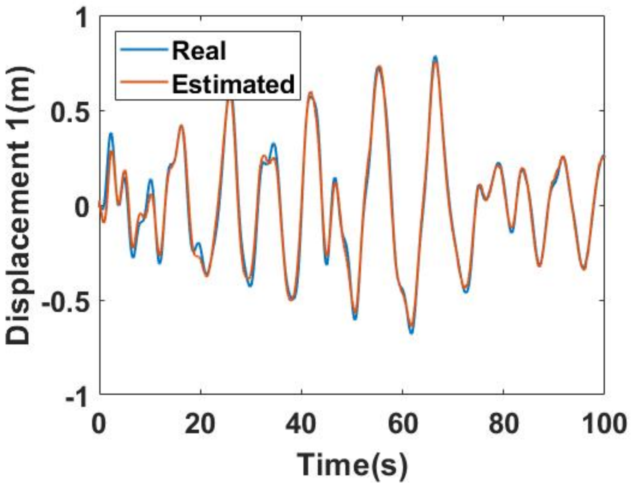

Fig. 3: The estimation of the displacement of buoy 1

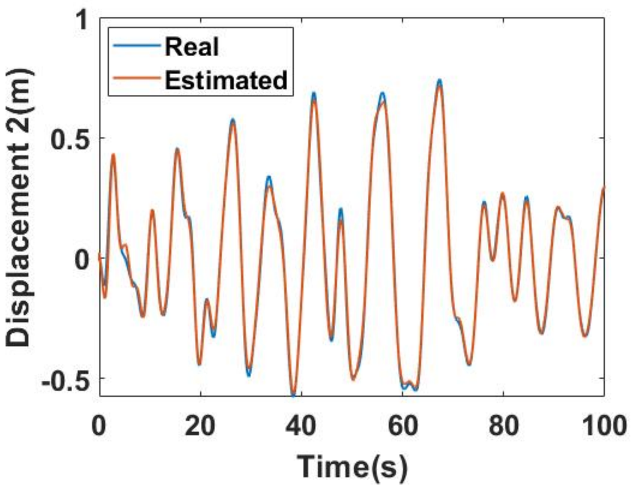

Fig. 4: The estimation of the displacement of buoy 2

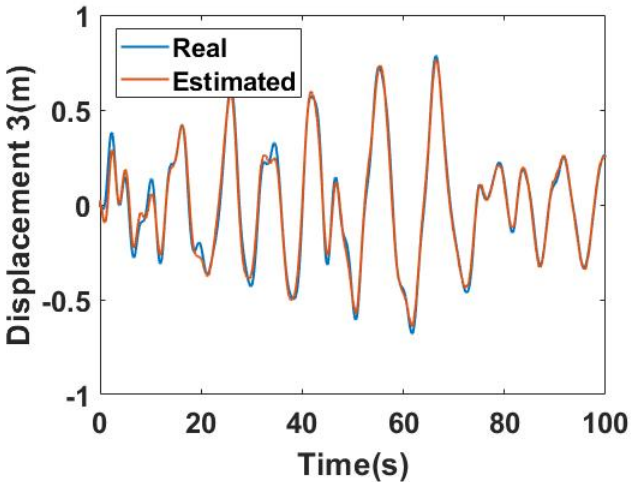

Fig. 5: The estimation of the displacement of buoy 3

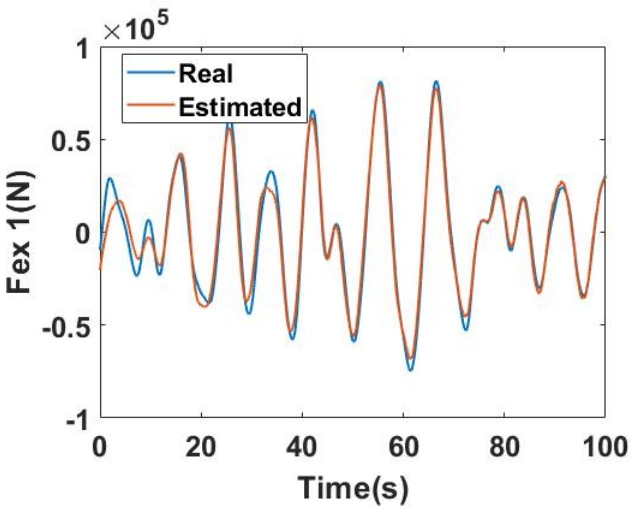

Fig. 6: The estimation of the Excitation force at buoy 1

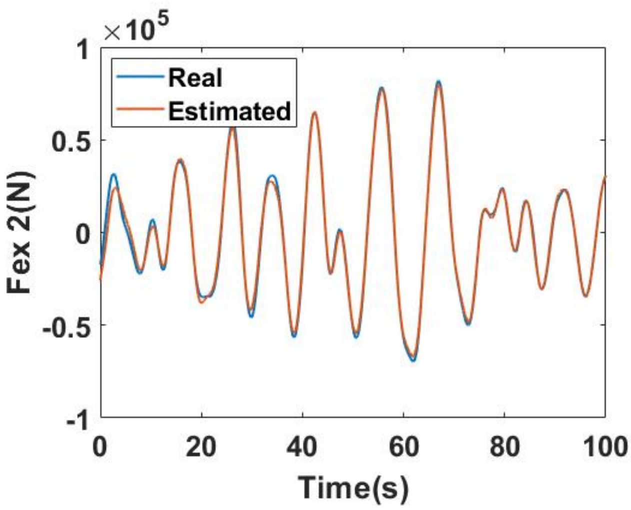

Fig. 7: The estimation of the Excitation force at buoy 2

adjusts the estimation based on local measurement without communicating the estimation of the other buoys. As shown in the figures, the estimation error of the KF is slightly better than CDKCF on buoy 1 and 3, while the CDKCF is significantly better than KF on buoy 2. Consequently, the bad estimation on buoy 2 will impact the performance of the control. In reality, the estimation performance of the agents in the array is unpredictable. However, with CDKCF, the estimation reaches consensus soon, the error in the estimation is 'averaged' and it will converge to 0. Additionally, as discussed before less number of frequencies is applied to estimate the wave field. However, as indicated in reference [28], with more number of frequencies, the performance of CDKCF will be improved more. Fig. 12 shows the CDKCF is significantly better than the KF in terms of disagreement.

B. The coupled forced WEC array

In this section, the performance of the CC with CDKCF is presented. The physical limitation on the control force is $u_{max} = 50$kN which is in the same level as the excitation force and on the displacement is $z_{max} = 0.8$m. The energy extraction of the WEC array is presented in Fig. 13. The average power over 100s is around 12.1kW compared to 12.8kW extracted by the ideal CC [21] which assumes perfect knowledge of the excitation force field. The control profile is shown in Fig. 14. The control force is saturated by the maximum

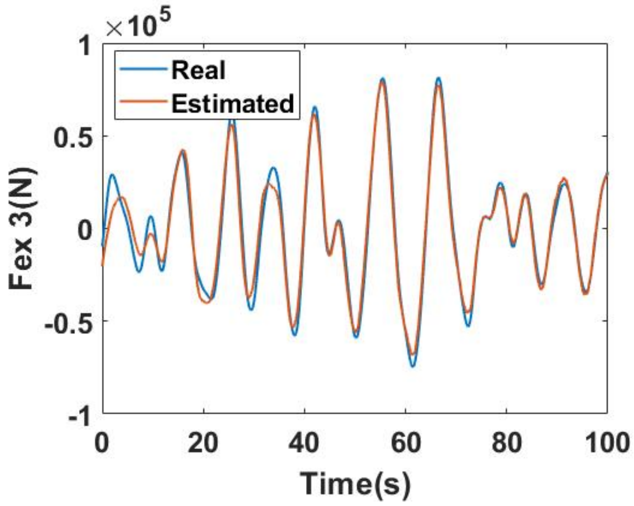

Fig. 8: The estimation of the Excitation force at buoy 3

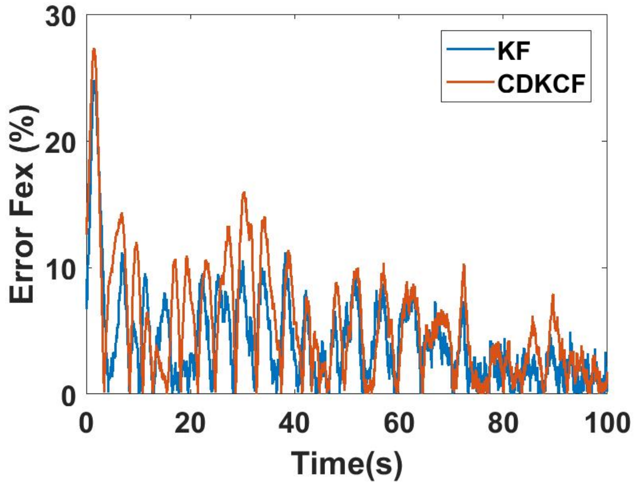

Fig. 9: The percentage estimation error of the excitation force

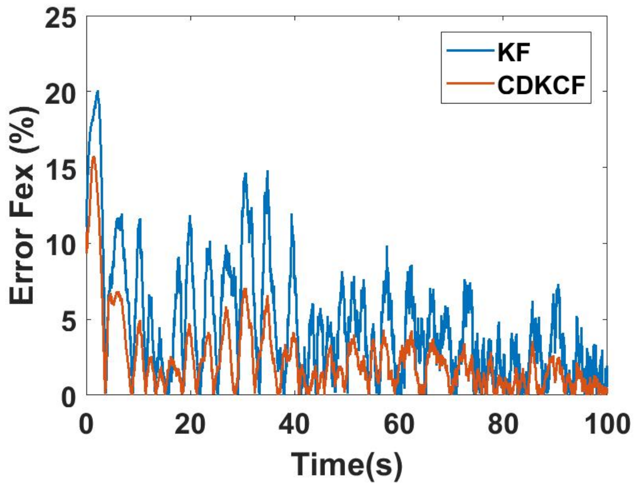

Fig. 10: The percentage estimation error of the excitation force

control capacity as shown in the figure. Fig. 15 shows the displacements of three buoys. From the figure, we can tell the displacements are all within the constraints.

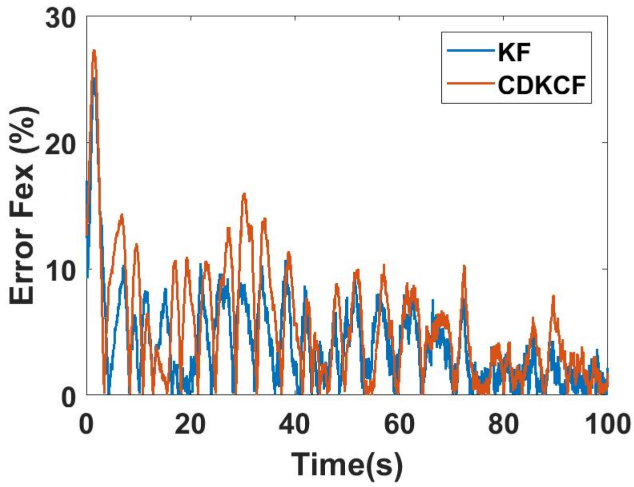

Fig. 11: The percentage estimation error of the excitation force

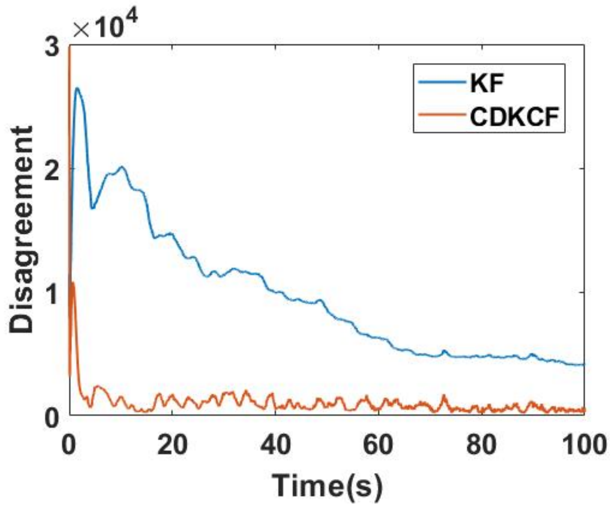

Fig. 12: The disagreement of the buoys in the array

Additionally, the motion of buoy 1 and 3 are almost identical which differs from the motion of buoy 2. The reason is the wave direction is 0^o (along the x-axis). Since buoy 1 and 3 have the same x-coordinate, they are expected to have identical motion. The average estimation error of the excitation force is shown in Fig. 16 for the coupled, forced WEC array. The estimation error converges towards 0%. Finally, the disagreement among different buoys in the array is shown in Fig. 17. The buoys in the array reach consensus within 20s.

IV. Conclusion

This paper introduces a novel adaptive Collective Control using the Continuous-Discrete Kalman Consensus Filter for the Wave Energy Converter array. The CC is implemented to maximize the energy extraction of the WEC array by considering the physical limitation on the control and displacement. To evaluate the control, the CDKCF is developed to estimate the required information. Since the hydrodynamics of the buoys are coupled in the array, the communication is desired to improve the estimation. The numerical simulations are conducted on the unforced and forced

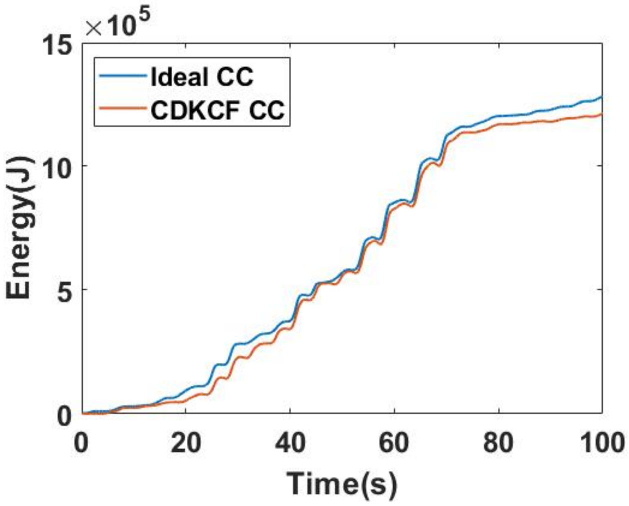

Fig. 13: The energy production of the WEC array

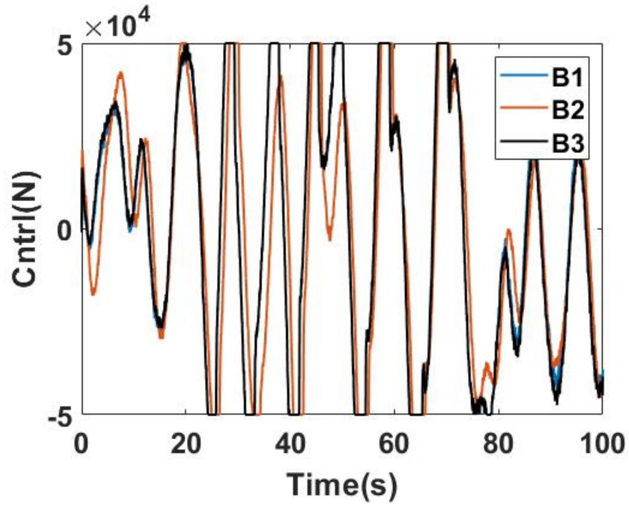

Fig. 14: The control force of three buoys

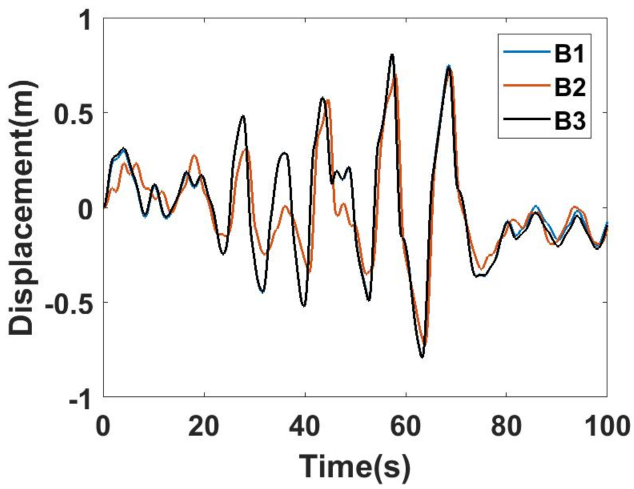

Fig. 15: The displacement of three buoys

WEC array. The simulation results of the estimation for the unforced WEC array indicates the CDKCF has an accurate estimation. Moreover, the CDKCF is better than regular KF both in terms of the estimation error and the disagreement. The performance of the adaptive

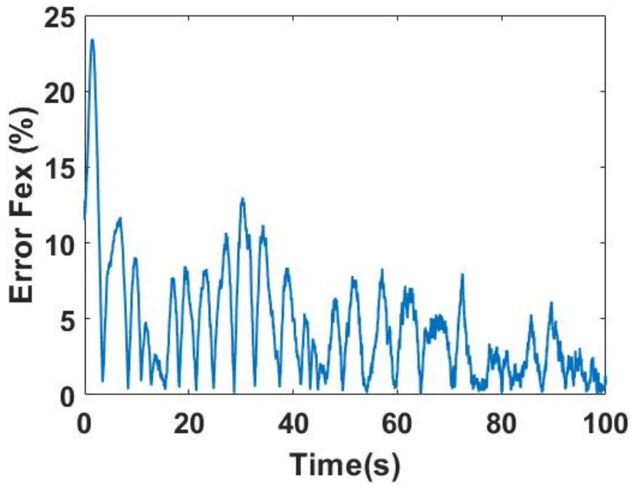

Fig. 16: The average percentage estimation error of the excitation force field

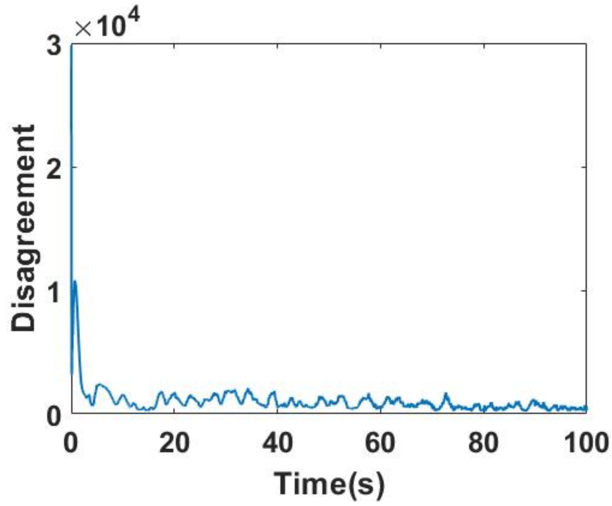

Fig. 17: The disagreement of the buoys in the array

CC is also simulated, the results indicate the proposed controller can optimize the energy extraction of the array and can extract 95% of the power produced by the ideal CC with CDKCF. In the future, the proposed control can be examined for an array with more buoys and the control law can be further improved both concerning energy production and computational efficiency.

References

[1] K. Budal, "Theory for absorbtion of wave power by a system of interacting bodies," *Journal of Ship Research*, vol. 21, pp. 248–253, 1977.

[2] D. V. Evans, "A theory for wave-power absorption by oscillating bodies," *Journal of Fluid Mechanics*, vol. 77, pp. 1–25, 9 1976. [Online]. Available: http://journals.cambridge.org/article_S0022112076001109

[3] C. C. Mei, "Power extraction from water waves," *Journal of Ship Research*, vol. 20, pp. 63–66, June 1976.

[4] J.N.Newman, "The interaction of stationary vessels with regular waves," in *Proceeding 11th symposium, Naval hydrodynamics*, London, 1976. [Online]. Available: http://journals.cambridge.org/article_S0022112076001109

[5] J. Falnes, *Ocean waves and oscillating systems: linear interactions including wave-energy extraction*. Cambridge university press, 2002.

[6] S. Zou, O. Abdelkhalik, R. Robinett, G. Bacelli, and D. Wilson, "Optimal control of wave energy converters," *Renewable Energy*, vol. 103, pp. 217–225, 2017.

[7] G. Li and M. R. Belmont, "Model predictive control of sea wave energy converters–part i: A convex approach for the case of a single device," *Renewable Energy*, vol. 69, pp. 453–463, 2014.

[8] G. Bacelli and J. V. Ringwood, "Numerical optimal control of wave energy converters," *IEEE Transactions on Sustainable Energy*, vol. 6, no. 2, pp. 294–302, 2015.

[9] O. Abdelkhalik, S. Zou, R. Robinett, G. Bacelli, D. Wilson, and R. Coe, "Control of three degrees-of-freedom wave energy converters using pseudo-spectral methods," *Journal of Dynamic Systems, Measurement, and Control*, vol. 140, no. 7, p. 074501, 2018.

[10] P. McIver, "Some hydrodynamic aspects of arrays of wave-energy devices," *Applied Ocean Research*, vol. 16, no. 2, pp. 61–69, 1994.

[11] H. Kagemoto and D. K. Yue, "Interactions among multiple three-dimensional bodies in water waves: an exact algebraic method," *Journal of Fluid mechanics*, vol. 166, pp. 189–209, 1986.

[12] T. Kinoshita, W. Bao, and S. Sunahara, "The hydrodynamic forces acting on a cylinder array oscillating in waves and current," *Journal of Marine Science and Technology*, vol. 2, no. 3, pp. 135–147, 1997.

[13] C. Lee, *WAMIT Theory Manual*, ser. Report (Massachusetts Institute of Technology. Department of Ocean Engineering). Massachusetts Institute of Technology, Department of Ocean Engineering, 1995. [Online]. Available: http://books.google.com/books?id=fWKGHAAACAAJ

[14] A. Babarit and G. Delhommeau, "Theoretical and numerical aspects of the open source BEM solver NEMOH," in *11th European Wave and Tidal Energy Conference (EWTEC2015)*, ser. Proceedings of the 11th European Wave and Tidal Energy Conference, Nantes, France, Sep. 2015. [Online]. Available: https://hal.archives-ouvertes.fr/hal-01198800

[15] B. Borgarino, A. Babarit, and P. Ferrant, "Impact of wave interactions effects on energy absorption in large arrays of wave energy converters," *Ocean Engineering*, vol. 41, pp. 79–88, 2012.

[16] A. De Andrés, R. Guanche, L. Meneses, C. Vidal, and I. Losada, "Factors that influence array layout on wave energy farms," *Ocean Engineering*, vol. 82, pp. 32–41, 2014.

[17] G. Bacelli, P. Balitsky, and J. V. Ringwood, "Coordinated control of arrays of wave energy devicesbenefits over independent control," *IEEE Transactions on Sustainable Energy*, vol. 4, no. 4, pp. 1091–1099, 2013.

[18] P. Balitsky, "Modelling controlled arrays of wave energy converters," Ph.D. dissertation, National University, 2013.

[19] D. Oetinger, M. E. Magaña, and O. Sawodny, "Decentralized model predictive control for wave energy converter arrays," *IEEE Transactions on Sustainable Energy*, vol. 5, no. 4, pp. 1099–1107, 2014.

[20] M. Starrett, R. So, T. K. Brekken, and A. McCall, "Increasing power capture from multibody wave energy conversion systems using model predictive control," in *Technologies for Sustainability (SusTech), 2015 IEEE Conference on*. IEEE, 2015, pp. 20–26.

[21] O. A. Shangyan Zou, "Collective control in arrays of wave energy converters," *IEEE Transactions on Sustainable Energy*, under review.

[22] H.-N. Nguyen and P. Tona, "Wave excitation force estimation for wave energy converters of the point-absorber type," *IEEE Transactions on Control Systems Technology*, 2017.

[23] O. Abdelkhalik, S. Zou, R. Robinett, G. Bacelli, and D. Wilson, "Estimation of excitation forces for wave energy converters control using pressure measurements," *International Journal of Control*, vol. 90, no. 8, pp. 1793–1805, 2017.

[24] M. Abdelrahman, R. Patton, B. Guo, and J. Lan, "Estimation of wave excitation force for wave energy converters," in *Control and Fault-Tolerant Systems (SysTol), 2016 3rd Conference on*. IEEE, 2016, pp. 654–659.

[25] M. Garcia-Abril, F. Paparella, and J. Ringwood, "Excitation force estimation and forecasting for wave energy applications," *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 14 692–14 697, 2017.

[26] R. Olfati-Saber, "Distributed kalman filtering for sensor networks," in *Decision and Control, 2007 46th IEEE Conference on*. IEEE, 2007, pp. 5492–5498.

[27] ——, "Kalman-consensus filter: Optimality, stability, and performance," in *Decision and Control, 2009 held jointly with the 2009 28th Chinese Control Conference. CDC/CCC 2009. Proceedings of the 48th IEEE Conference on*. IEEE, 2009, pp. 7036–7042.

[28] S. Zou and O. Abdelkhalik, "Consensus estimation in arrays of wave energy converters," *IEEE Transactions on Sustainable Energy*, 2018.

[29] W. Cummins, D. T. M. B. W. D. C., and D. W. T. M. Basin, *The Impulse Response Function and Ship Motions*, ser. Report (David W. Taylor Model Basin). Navy Department, David Taylor Model Basin, 1962. [Online]. Available: https://books.google.com/books?id=GLLANwAACAAJ