Wave Spectral Analysis for designing Wave Energy Converters

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Abstract—**Assessing wave energy for estimating the resource potential, and further for determining design and operational parameters, requires a deep understanding of the local wave spectral characteristics. The analysis based on bulk *Hs* and *Tp* parameters only is not sufficient. Here we present a spectral statistical approach that allows identifying the main spectral components and their characteristics. Among others, the spectral band width, seasonal variability, and extremes play a crucial role in the actual potential. Complementary, for a quick global view using integral parameters, it is observed that the average bulk wave power is misleading, other parameters offer a more objective perspective. Based on that, we find that equatorial regions rather than extra-tropical ones are more attractive for wave energy exploitation.**

Keywords—**Spectral statistics, wave spectral partitioning, WEC design parameters.**

I. Introduction

WAVE energy is an attractive, although yet untapped, renewable resource for potential human utilization. Attractive because our current estimates suggest that the available magnitudes are significant, and compatible with (or even grater than) those from other renewable sources (such as wind and solar) [e.g., 1]. In addition, synergies immediately arise, as for instance, offshore wind and wave farms are fully compatible in terms of space utilization, connection to the supply grid, and installation and maintenance procedures [e.g., 2]. On the other hand, our potential ability to reduce or control the wave power reaching the coasts serves the purpose of controlling sediment transport patterns, which is a widespread problem affecting all coastal regions [e.g., 3].

Nevertheless, there are also significant challenges associated to this energy source. Among them, the environmental impact should be first in the list, because our current diversification of energy sources with emphasis in renewable ones responds to the global dangers, increasingly more notorious, related to our indiscriminate use of fossil and nuclear sources [e.g., 4]. Unfortunately, progress reluctantly works that way, as technologies are first put in place, and then assessed environmentally [e.g., 5]. In any case, technological barriers (rather than environmental) are the ones currently prohibiting us the use of this source. A conspicuous limitation is that of materials, since it is pretty apparent that the converting device will have to stand strong forces, while interacting actively with salt water. That makes stainless steel, dubiously, our only affordable option, or a better candidate material yet to be invented. Although the naval and offshore industries are well ahead in this field, the contact of moving parts with salt water in current applications, tends to be avoided as much as possible [e.g., 6].

However, there are other less conspicuous, but probably more severe challenges, related to the hydrodynamic interaction of the would-be wave energy converter (WEC), and waves. The first is the frequency incompatibility between ocean waves and those of (e.g.) floating devices, the former in the order of e.g., 1/10 Hz, and the later much higher in the order of e.g. ½ Hz [e.g., 7]. While resonating at its natural frequency the WEC will capture energy of waves at similar frequencies, but regarding waves, that frequency (½ Hz) corresponds to the tail of the wave spectrum, where only a marginal amount of the total energy resides. This is not a trivial technical challenge because (e.g.) in the spirit of a mass-spring-damper system, lowering the natural frequency translates into inoperable masses or forces in the mechanical components [e.g., 8]. Nevertheless, this remains an engineering problem, sooner or later to be overcome.

Another issue to take into account is the energy distribution and variability at single location, referring to time, magnitude, and spectral space. When we talk about time and magnitude, we refer mainly to seasonality, because the extra-tropical storm generating zones, that usually reflect the largest bulk average wave power, suffer from large seasonal variability, with little activity in summer, and potentially destructive conditions in winter [e.g., 9]. In turn, when we talk about spectral

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distribution, we refer to how much wave energy is dispersed in frequency and direction, or in more simple terms we are talking about wind-sea and swell. Clearly, if the oscillatory input force comes in the form of a single, unidirectional, sinusoidal wave, it is easier to harvest than if it comes from multiple directions and sizes. Most conceptual designs are based on this idea of a pristine sinusoidal wave [e.g., 10]. However, at the generation location wind waves are born naturally in a very chaotic form, they brake and crash into each other constantly [e.g., 11]. The second law of thermodynamics dictates that the more disorganized energy is, the less its quality, and the more difficult to convert it to work. As far as the authors know, a quantification of a possible upper limit for wave energy conversion efficiency as a function of spectral dispersion has not been established yet, but our second law intuition tells us it must exist. In that spirit, we believe that understanding the spectral energy distribution is the paramount step, not only for properly designing a WEC, but also to know what sector of the spectrum we are aiming for, how much energy is available in that sector, and how much of that energy can actually be harvested. This aspect has been previously explored [in e.g., 9], but in the mean time more robust methodologies have been developed to characterize wave spectra time series, allowing the identification of wave systems (or wave families) in a climatic sense (e.g., [12]). In this work, we make use of the concept of wave systems to gain insight into the resource available. We believe this knowledge to be valuable not only for defining WEC's design parameters, but also for facilitating its design as such, to draw a more realistic expectation of the conversion efficiency, and potentially also to facilitate its operation.

II. Data and Methods

A. Data

In the present work we use model data from the ECMWF (European Centre for Medium-Range Weather Forecasts), from both the ERA-I [13], and ERA-5 [14] archives. Integral parameters for global maps are derived from ERA-I, while spectral data corresponds to ERA-5. These data sets are described in detail elsewhere, so for the sake of compactness, we refer the reader to the given references for further details.

B. The wave power equation

Wave power consists of oscillating kinetic and potential components of gravitational waves traveling on the water surface. The basic equations for wave dynamics, based on linear (i.e., sinusoidal) theory assume an infinitely wide wave (in the direction perpendicular to the travel direction). However, the wave spectrum as implemented in models and measuring devices, is single point based. Therefore, when we compute wave power using single point data we end up with the rather uncomfortable units of power per unit length. That is, energy flux per meter of wave width.

Wave energy travels at the speed of the group, so wave power is computed as the product of energy and speed as expressed in (1) (e.g., [15]). That value corresponds to power along the wave direction, so in that equation, we neglect directionality "to our advantage", assuming that all directions contribute to our purposes. However, this is not the case, the energy within the spectrum is distributed along several directions, the more so, the closer the waves are to the generation zone (wind-sea).

$$P_{\text{energy}} = \mathrm{E}(f) * c_g(f) \quad (1)$$

The group speed in deep water is given by (2):

$$c_g = \frac{g}{4\pi \mathrm{f}} \quad (2)$$

where f is the wave frequency in Hz, and g is the gravity acceleration. For a composite of several sinusoidal waves (i.e., the wave spectrum), E is also frequency dependent, so the total wave power is obtained as (3):

$$P_{\text{energy}} = \rho \mathrm{g} \int_0^\infty E\,(f) c_g(f) \quad (3)$$

Introducing (2) in (3), and working out the algebra in terms of parameters more familiar to us (i.e., *Hs* and *Tm*), we obtain the more widely used expression (4):

$$P_{\text{energy}} = \frac{\rho \mathrm{g}^2}{64\pi} H_s^2 T_{m-1,0} \quad (4)$$

Which gives us the total power available, or rather its upper limit. However it is important to bear in mind that intrinsically not all that energy can be converted into a mechanical form, because (among others) the aforementioned constraints, e.g., all waves are not traveling consistently in the same direction, as implicit in (4). This is one of the reasons justifying the need of insight into the spectral characteristics, to answer questions like: how are local waves distributed in direction? How are they distributed in frequency?, and how all this affect the available wave power? Clearly, the usual scatter diagram *Hs-Tp* does not contain such information, because by computing those parameters we neglect the energy distribution in frequency and direction.

III. Spectral versus Integral Parameters

Therefore, when designing a WEC it is inappropriate to consider overall bulk parameters (e.g., the typical scatter diagrams *Hs-Tp*). The reason is that in most of the world, wave conditions are complex and involve several long-term wave systems, each with different characteristics. This is true even for apparently simple basins like the Mediterranean or the North Sea, in the open ocean this is certainly the case with swells arriving from remote places. For our computations we focus in the eastern equatorial Pacific Ocean, more specifically in the vicinity of the Galapagos archipelago. In order to gain insight into the spectral distribution we follow a spectral statistics approach (e.g., [12]). According to this approach, a long time-series of spectra can be characterized by studying the distribution of spectral components (or partitions). For a single time step spectrum, a wave component is identified as a self consistent cluster of spectral bins using a partitioning algorithm (e.g., [9]). Physically, a wave partition represents a single event, originated in turn by a single meteorological event. Consequently, it is found that the long term distribution of partitions point out to climatic features, that are typically few and well defined at each location. To further illustrate this concept and analyze the local characteristics in detail, Fig. 1, shows the main spectral statistical indicators at the study location (91.08⁰W, 1.8 ⁰S). In addition, Fig. 2 shows the traditional *Hs-Tp* scatter diagram for comparison.

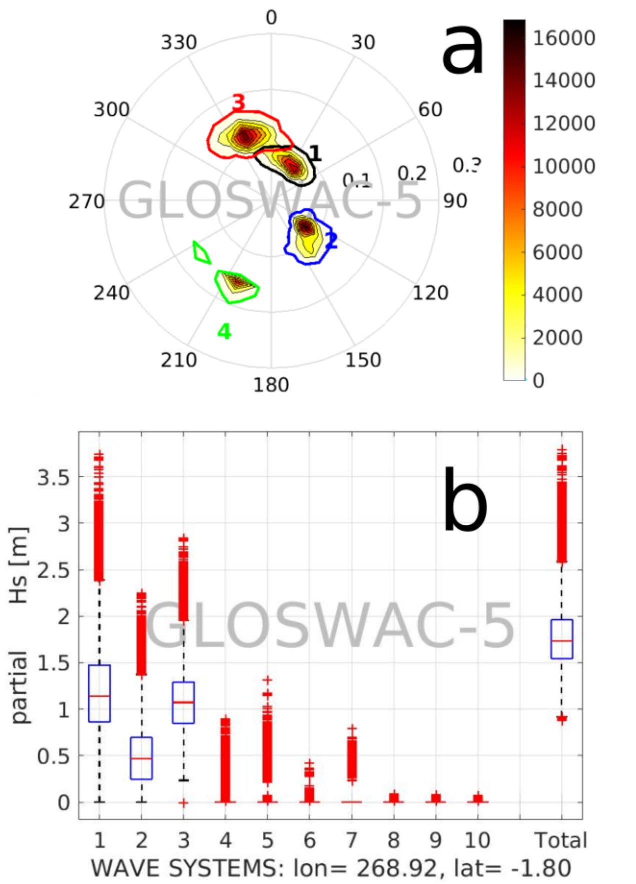

Fig. 1. Wave spectral statistics at the study location (91.08⁰W, 1.8 ⁰S). a) empirical distribution of partitions showing the different wave systems present (directions shown are 'going to'), b) box-and-whisker plot of *Hs* for the different wave systems.

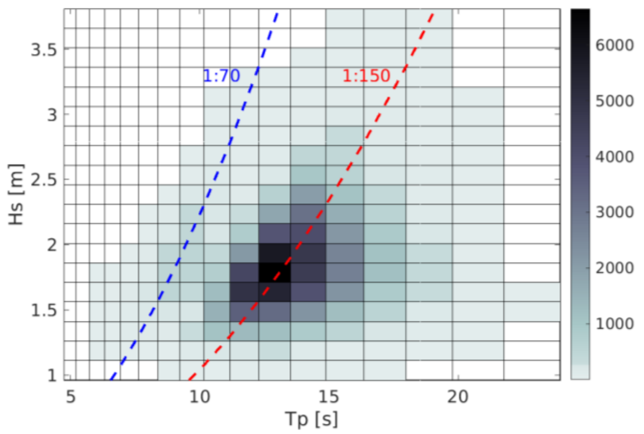

Fig. 2. Scatter diagram *Hs-Tp* at 91.08⁰W, 1.8 ⁰S, colors show number of occurrences, dashed lines correspond to values of constant wave steepness.

Both Figures 1, and 2 try to convey the same information, which is the long terms characteristics of local waves. The difference is that the indicators in Fig. 1 preserve the information embedded in the spectral time series. Fig. 1a shows the long term distribution of partitions, where the colors and color bar indicate recurrence. The main wave systems can be identified as the clusters within this distribution (numbered from 1 to 4), which in turn can be associated to their meteorological origin. In Fig. 1 we readily identify four wave systems, although the algorithm is set to identify a maximum of ten wave systems, from the fifth on, at this location, they are rather marginal in terms of occurrences and also in terms of magnitude, so they will not be considered for the present analysis. Looking at directional information only, it is clear that these four wave systems have different origin, WS1 and WS2 come from the southern hemisphere, while WS2 and WS4 come from the southern hemisphere. Apart from that, there is a clear separation among most of them, with overlapping (e.g., possibly ambiguous) limits only between WS1 and WS3. Background knowledge of the global meteorological conditions, plus the characteristics of the distributions in Fig. 1a themselves, let us infer the following: WS1 corresponds to swells originated in the southern Pacific Ocean, most of them generated as far as in the Antarctic storm belt. These swells are ubiquitous in the southern oceans because the generation zone covers the entire circle and radiates wave energy primarily in the NE direction. We know it is swell because of the typical frequencies, all below 0.1 Hz. Similarly, WS2 is also a swell system, in this case originated in the northern hemisphere. The main source of this wave system are the extra-tropical storm zone of the Pacific Ocean, although there is also significant activity along the Baja California peninsula and the south of Mexico. In turn, WS3 is produced by the southern trade winds active all along the South American Pacific coast. At the study location the fetch of WS3 is rather long, and these waves are slightly beyond the actual generating zone, so they classify better as old wind sea rather than purely wind sea. Finally WS4 is due to the wind jets crossing Central America, particularly at this location we see the branch belonging to the Papagayo wind jet, which at the considered location has a minor influence, so we will not consider this wave system any further for potential energy harvesting.

Note that Fig. 1a contains information of frequency and direction only, so this information needs to be complemented with the energy magnitude, or *Hs*, which is more familiar to us. In this approach, *Hs* is given in a separate plot, in the statistical form of a box and whiskers plot. Recall that the box-wishker representation is basically a simplified histogram. This presentation allows us to see how *Hs* is distributed for each of the wave systems. We readily observe their median values (red lines), their inter-quartile ranges (blue boxes), and the extremes (red crosses). Here we observe that WS1, WS2, and WS3 are the more energetic, with the rest of WSs contributing only marginally to the total energy. We observe also that above 2.5 m, conditions at this site already classify as extremes, which for extra-tropical standards sounds significantly modest.

We contrast the information of Fig. 1 with that of Fig. 2, which depicts the traditional *Hs*-*Tp* scatter diagram at the same location. In this diagram we observe that waves with about *Hs*= 1.8, and *Tp*= 13 s, are the most recurrent, we can derive some other characteristics related to these parameters' distribution, but essentially not much more than that. So in this study we advocate for the spectral statistical approach available in Fig. 1. We believe that any WEC's designer is better equipped with this information for carrying out his/her task.

IV. SPECTRAL CHARACTERISTICS AND SEASONAL VARIABILITY

Having identified the main active wave systems, it is also possible to extract their time series from the total and derive other relevant characteristics. Particularly important are the spectral characteristics and seasonal variability presented in Fig. 3.

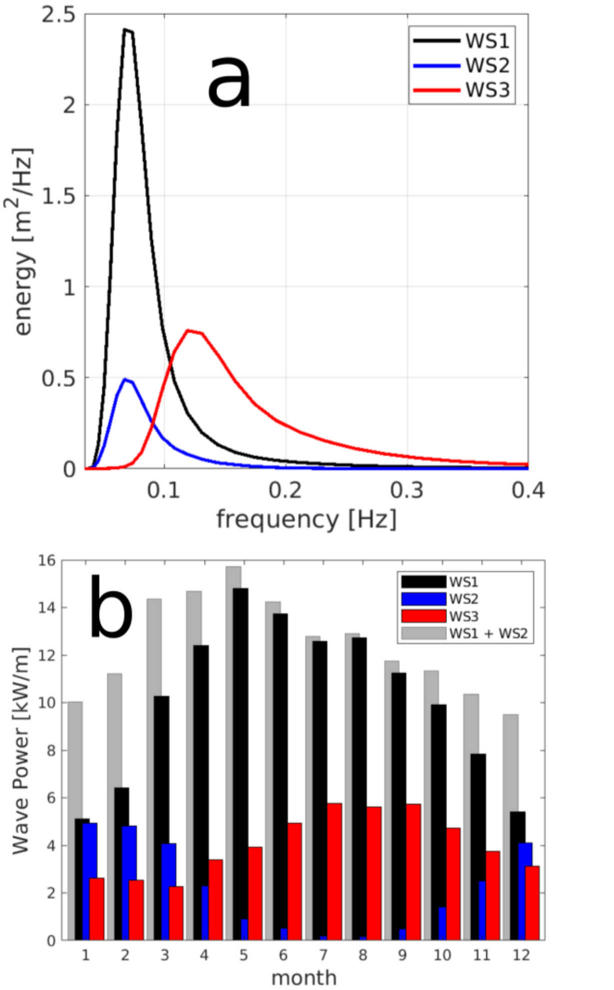

Fig. 3. a) Averaged 1D frequency spectrum and b) Mean seasonal wave power, for the three main individual wave systems detected at the study location.

In both panels we observe that WS1 is indeed the most energetic from the tree of them, followed by WS3. As WEC designer one has to focus mainly in the peak of the spectral distribution, because this region is the more energetic one. In that regard, Fig. 3a shows that WS2 is very compatible with WS1 in terms of characteristic frequencies, it is indeed even narrower than WS1, although less energetic. Contrarily, WS3, although second in energy content, it does not share similar characteristics with WS1 and WS2. Its frequencies are higher, and the spectral distribution is wider. It is to be expected that the WEC's oscillating frequencies will not be that adaptable, particularly at the early stages of this technologies, so this parameter will have to be fixed. If we aim for the most energetic part of the spectrum, that is about 0.07 Hz. A slight increase of that oscillating frequency to 0.12 Hz will allows us to capture energy from WS3, but in terms of WS1 and WS2 we will be operating at the tail of the corresponding spectra where there is a minimum amount of energy. This means that at this location, using a non directional device, we can aim at the energy of WS1 and WS2, but we have to give up the energy of WS3. With this idea in mind, we present in Fig. 3b the seasonal variability of wave power for these tree WSs. Since we identify a potential to complement both WS1 and WS2 (with a non directional device), we have also plotted their combined magnitude (in gray). In this graph we clearly observe the seasonality of these WSs, highly dependent on the, austral and boreal, winter seasons respectively, so that WS1 and WS2 are out of face, but together they contribute to a more stable signal in yearly bases.

V. THE ROLE OF SEASONALITY AND EXTREMES

Although at first glance the equatorial region is not the most attractive in terms of global power as computed by (4), and averaged over decades, other considerations suggest it has indeed a remarkable potential. This apparent paradox arises because when assessing the magnitude of an environmental variable we look at statistical parameters, among which the mean is readily preferred, but seasonality and its associated extremes play a crucial role. To illustrate this, we revisit the work of Barstow et al., [16], who presented a series of conspicuous maps which, large and by, contain the relevant message. We reproduce that kind of maps here because we consider them a key for understanding the global potential of wave power. For point evaluation, [9] presented an approach for estimating operational parameters related to capacity factor, installed power, and net energy production. These tree parameters bear some dependence among them, while embedding also environmental characteristics. The nominal installed capacity is directly related to the energy production, but also to the installation costs. The capacity factor, is the fraction of time (out of a year unit) the WEC will produce energy at its nominal power, and therefore it is closely linked to seasonality. In addition, installation costs depend to the conditions to be withstood by the operating WEC, and therefore related to the extreme values.

The spectral analysis just presented, or a further analysis based on operational parameters (as presented in [9]) are difficult to present in the form of global maps, however, thematic maps based on integral parameters as those presented in Fig. 4, convey a good portion of the relevant information, if we keep in mind the associated limitations. Panel (a) shows the typical long term average of wave power, suggesting that the extra-tropical zones contain the highest potential. In turn, panels (c) and (d) show the average of the seasonally opposite months January and July respectively, offering a direct glance into the seasonal variability. Clearly the northern hemisphere displays a large activity during the boreal winter (January), but very little activity during its corresponding summer. The same is true for the southern hemisphere, although some activity prevails during its summer (January), because of the geographical characteristics. Panel (e) presents the ratio between the monthly minimum and the mean. This parameter is related to the capacity factor, because the closer to the mean, the more regular the output is to be expected.

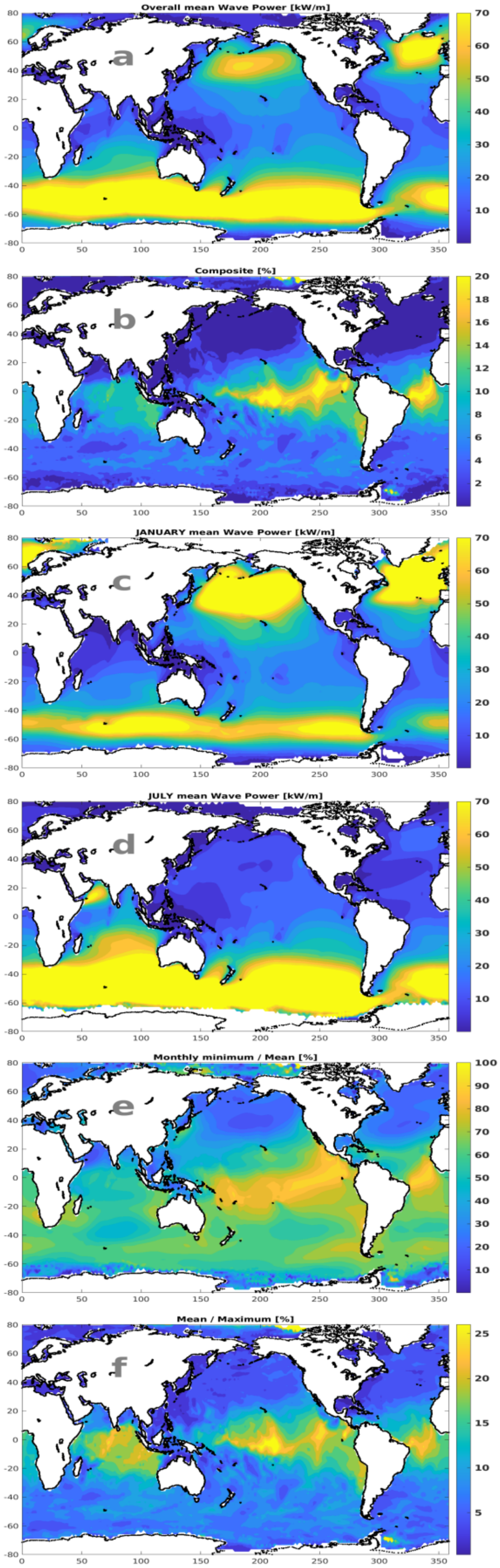

Fig. 4. Thematic maps of the global wave energy resources. a) mean global wave power, b) combined (intersection) of parameters in e and f. c) January monthly mean, d) July monthly mean, e) ratio

between the minimum monthly mean and the overall mean power, f) ratio between the overall mean and the maximum wave power.

This parameter is crucial to establish the economical viability of the project, and by itself, it reverses the perspective provided by panel (a), suggesting that the tropical region is more attractive than the extra-tropical ones. Further on, panel (f) shows the ratio of the mean to the maximum wave power, which is related to installation costs versus installed capacity. The lower the ratio, the larger the extremes compared to the potential (mean), therefore the higher the risk of damage, failure, or catastrophic loss. This panel too marks the equatorial zone as potentially more attractive. For a more objective look one has to consider the information of panels (e) and (f), preferable also in view of panel (a), so in panel (b) we have computed a parameter related to the intersection between (e) and (f), simply by multiplying these two parameters and normalizing the result. We note that other possibilities exist for this combination, but for a general global view the one presented is an indicative one. Conspicuously, the patterns of panel (b) are more related to the ITCZ (Inter-tropical convergence zone) than to the extra-tropical regions. This result that appears counter-intuitive at first glance, is fully justified in the light of the spectral parameters just described at the single (tropical) location.

VI. Conclusion

Understanding the wave energy resource requires insight into the spectral distribution of energy at each location. Here we present an approach based on spectral statistics, which allow to identify wave families (populations), and from there we can anticipate their meteorological origin and other fundamental characteristics. All this information being essential for WEC design and operation.

Global maps of averaged bulk wave power are misleading for assessing the most promising harvesting sites. Those maps suggests that the extra-tropical regions are the more interesting ones, while other more objective parameters, suggest precisely the opposite.

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